Journal of Physics Special Topics

An undergraduate physics journal

A1_5 Methane Mars 🐄💨💨

Y. Davis, N. Drewes, T. Lomax, S. Norman

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 9, 2020

Abstract

Here, we consider the boundary to space - the Kármán line - of a hypothetical methane atmosphere on Mars with a surface pressure of 1 atm. To do this, we derived expressions for the particle density where the lift of a plane can no longer increase, and compared it to the particle density on Mars as a function of height through the Law of Atmospheres. This gives a Kármán line at ~ 210 km.

Introduction

If magical cows on Mars were burping and farting to fill up a methane atmosphere, how much methane would they need to produce? Well, first we need to define the edge of the atmosphere, one definition of which is the Kármán line. This method considers a plane flying in the hypothetical Mars atmosphere and finds the height at which the lift of the plane is equal to the force of gravity acting on it. To do this, we will assume an Earth-like atmosphere with a pressure of 1 atm at the surface and obtain an expression for the lift of a plane, which will then be compared to the density of methane gas as a function of altitude through the Law of Atmospheres.

Theory

The Kármán line is defined as the point at which a plane travelling at the orbital velocity can no longer achieve any lift [1]. Therefore, the upward force (the lift), L on a plane of mass m, travelling at orbital velocity v_0, is equal to the force of gravity on that plane

$$L = \frac{1}{2}\rho v_0^2 S C_L = m g_M \tag{1}$$

where ρ is the density of the gas the plane is travelling through, S is the wing area, C_L is its lift coefficient and g_M is the acceleration due to gravity on Mars. The orbital velocity at a height y above the surface of Mars is found by equating the centripetal and gravitational forces, and is

$$v_0^2 = \frac{GM_M}{(R_M + y)} \tag{2}$$

where G is the universal gravitational constant, and M_M and R_M are the mass and radius of Mars respectively. If we introduce the definition of $g_M = GM_M/R_M^2$, then v_0^2 can be written as

$$v_0^2 = \frac{g_M R_M^2}{R_M + y}. \tag{3}$$

Substituting that back into the definition of the Kármán line, and solving for ρ gives

$$\rho = \frac{2m(R_M + y)}{R_M^2 S C_L}. \tag{4}$$

The Law of Atmospheres is the pressure, P, at a height y above the surface, given by

$$P = P_0 \exp\left(-(\rho_0/P_0) g_M y\right) \tag{5}$$

where ρ_0 and P_0 are the density and pressure at the surface respectively [2]. Assuming that the particles of methane act as an ideal gas, we may use the ideal gas law, $PV = NkT$, where N is the number of particles, V is the volume of gas, and T is the temperature. Therefore, we can re-arrange Equation 5 for the particle number density, n as a function of height

$$n = \frac{P_0}{kT} exp(-(\rho_0/P_0)g_M y). \tag{6}$$

Multiplying both sides by the particle mass of methane, m_p, the mass density ρ is

$$\rho = \frac{m_p P_0}{kT} \exp\left(-(\rho_0/P_0)g_M y\right). \tag{7}$$

Finally, by re-arranging the ideal gas law for $n = P/V$, and multiplying by the mass of a methane particle, we get

$$\rho_0 = \frac{m_p P_0}{kT}. \tag{8}$$

Substituting this for ρ_0 in Equation 7, we obtain an expression for the particle density as a function of height that we can calculate

$$\rho = \frac{m_p P_0}{kT} \exp\left(-(m_p/kT)g_M y\right). \tag{9}$$

Results

Graphing both Equations 4 and 9 (assuming a constant temperature profile for simplicity) and evaluating the x-coordinate of their intersection will give the height of the Kármán line. Values for constants are $g_M = 3.72\,\mathrm{m\,s^{-2}}$, $R_M = 3380\,\mathrm{km}$, $P_0 = 1\,\mathrm{atm} \sim 1 \times 10^5\,\mathrm{Pa}$, $T = -60\,°\mathrm{C} = 213\,\mathrm{K}$, $m_p = 2.664 \times 10^{-26}\,\mathrm{kg}$, and for an A320 aeroplane, $S \sim 120\,\mathrm{m}^2$, $m \sim 80\,\mathrm{t}$ and $C_L \sim 0.5$ [3, 4]. Figure 1 shows a plot of the two lines over a range of heights from 0 - 300 km, where the blue line is Equation 4 and the red line is Equation 9. From this plot, the Kármán line is at $y \sim 210\,\mathrm{km}$.

Conclusion

Overall, we found that for a hypothetical methane atmosphere on Mars that acts as an ideal gas and has a surface pressure of 1 atm, the Kármán line is at $\sim 210\,\mathrm{km}$. This is over twice as high as the Kármán line on Earth at 100 km [1]. However, due to the decreased gravity, the scale height in the Law of Atmospheres increases on Mars, which means that the density decreases at a slower rate.

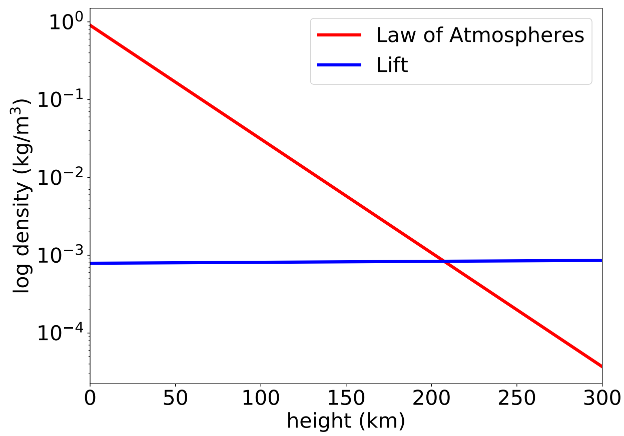

Figure 1: A plot of the particle density required for a plane to reach maximum lift (blue), and that of a hypothetical Mars atmosphere with increasing height (red). The two lines intersect at $\sim 210\,\mathrm{km}$.

References

[1] `https://metro.co.uk/2017/08/29/how-high-is-space-and-what-is-the-karman-line-6886681/` [Accessed 23 November 2020].

[2] *Physics for Scientists and Engineers*, P.A. Tipler and G. Mosca, Freeman, New York, 2007. P.431.

[3] `https://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html` [Accessed 23 November 2020]

[4] `https://en.wikipedia.org/wiki/Airbus_A320_family` [Accessed 23 November 2020]