Journal of Physics Special Topics

An undergraduate physics journal

P3_5 Fly Me To a Stop

O. D. M. Chatwin, J. D. Holtom, A. Iorga, and S. Tyler

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 3, 2020

Abstract

We investigate the effect of Voyager 1's gravity assist (or fly-by) on the planet Jupiter. We consider the interaction as an elastic collision and use the conservation of energy and momentum to calculate Jupiter's change in velocity following the fly-by. We determine that after each gravity assist the velocity decreases by $2.229 \times 10^{-23}\%$ and therefore, after 4.350×10^{24} gravity assists Jupiter's velocity will reach $0ms^{-1}$.

Introduction

Voyager 1 was launched by NASA in 1977 [1] with the primary mission of exploring the outer solar system. In order to reach the edge of our solar system it completed two gravity assists; one with Jupiter and one with Saturn. By 1979 [1], Voyager 1 had reached Jupiter and completed it's fly-by.

The fly-by of Jupiter gave Voyager 1 the additional energy and momentum required to travel further through the solar system. This energy and momentum came from the planet Jupiter itself, and so the fly-by must have had an affect on the planet, however small.

In this paper, we calculate the number of gravity assists necessary to reduce the velocity of Jupiter to $0ms^{-1}$.

Theory

A gravity assist is used to accelerate a space craft and conserve fuel. A space craft travels close enough to a planet to be influenced by its gravity. If the space craft flies in the same direction as the planet, it gains some of the planets energy and momentum.

When a gravity assist occurs, there is a transfer of energy and momentum. Since both must be conserved, when energy and momentum is transferred from the planet to the space craft, the planet will lose some energy and momentum to the craft. The planets velocity will therefore decrease, relative to the sun.

The interaction itself mimics that of an elastic collision [2], therefore it is possible to calculate the velocities of both bodies after a gravity assist.

Results and Discussion

The conservation of momentum and energy are given by equations 1 and 2 respectively.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{1}$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2 \tag{2}$$

Where m_1 is the mass of Voyager 1 [3], m_2 is the mass of Jupiter [4], u_1 and v_1 are the respective velocities [5] of Voyager 1 before and after the gravity assist, and u_2 and v_2 are the respective velocities of Jupiter [4, 6] before and after the gravity assist.

Equations 1 and 2 can be rearranged to determine values of v_1 and v_2.

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2 \tag{3}$$

$$v_2 = \frac{2m_1}{m_1 + m_2} u_1 + \frac{m_2 - m_1}{m_1 + m_2} u_2 \tag{4}$$

The above equations give the velocity of both bodies after the gravity assist.

Using $m_1 = 733kg$, $m_2 = 1.898 \cdot 10^{27} kg$, $u_1 = 17000ms^{-1}$ and $u_2 = 13100ms^{-1}$, equation 5 is used to calculate the change in velocity of Jupiter after the gravity assist, v', and is equal to $3.012 \times 10^{-21} ms^{-1}$.

$$v' = v_2 - u_2 \tag{5}$$

v' is $2.299 \times 10^{-23}\%$ of u_2, therefore with each gravity assist Jupiter's velocity reduces by $2.299 \times 10^{-23}\%$. Using the percentage difference rather than the absolute value accounts for the change in v' as Jupiter loses energy following each fly-by. This means that in order to reduce Jupiter's velocity to $0ms^{-1}$, Voyager 1 would need to make 4.350×10^{24} gravity assists.

Conclusion

When a gravity assist occurs, the momentum and energy of both the craft and the planet must be conserved. During Voyager 1's fly-by of Jupiter, Jupiter's velocity reduced by $3.012 \times 10^{-21} ms^{-1}$, and therefore after 4.350×10^{24} gravity assists, Jupiter's velocity will be $0ms^{-1}$. Luckily, it is unlikely that a Voyager 1 type craft will make this many fly-bys of Jupiter in the near future and therefore we do not need to be concerned about the chance of the planet stopping altogether.

References

[1] `https://voyager.jpl.nasa.gov/frequently-asked-questions/fact-sheet/` [Accessed 12 November 2020]

[2] `https://en.wikipedia.org/wiki/Gravity_assist` [Accessed 12 November 2020]

[3] `https://www.nasa.gov/centers/goddard/news/topstory/2003/1105voyager_facts.html` [Accessed 12 November 2020]

[4] `https://solarsystem.nasa.gov/planets/jupiter/by-the-numbers/` [Accessed 12 November 2020]

[5] `https://voyager.jpl.nasa.gov/frequently-asked-questions/fast-facts/` [Accessed 12 November 2020]

[6] `http://spiff.rit.edu/classes/phys311/workshops/w10b/pluto/pluto.html`[Accessed 28 November 2020]