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P2_3 Spinning Asteroid Habitats

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Abstract

The concept of using centripetal acceleration from circular motion as a source of artificial gravity is a known possible solution to surviving weightless environments long term. We applied this concept to asteroids, calculating that it would take of the order 10^{20} J to spin up an asteroid equivalent to Phobos in order to make the centripetal acceleration equal to 9.81 ms^{-2}.

Introduction

When considering the habitability of the inner solar system, the discussion is usually limited to Earth, the Moon, and Mars. However, there is a concept whereby an asteroid could be spun-up to a high enough rotation speed that it would be possible to build structures on the surface with centripetal acceleration keeping you pinned to the ceiling (living upside down relative to the asteroid). In this paper, we take a whistle-stop tour of the concept, exploring the mechanics of the rotating asteroids, the energy required to get them rotating at sufficient speeds, and the plausibility of the idea for future habitation.

Theory

The key equations we needed for our calculations are standard equations used for angular/circular motion. The equation relating centripetal acceleration (a_c) of a body in circular motion to its angular velocity ω and radius r [1]:

$$a_c = \omega^2 r = \left(\frac{\Delta\theta}{\Delta t}\right)^2 r \tag{1}$$

For a spherically symmetric body of uniform density with mass M and radius r, the moment of inertia I is [1]:

$$I = \frac{2}{5} M r^2 \tag{2}$$

The rotational energy E stored in a rotating body with a moment of inertia I and angular velocity ω is given by [1]:

$$E = \frac{1}{2} I \omega^2 \tag{3}$$

Method and Results

We decided to choose a small near-spherical asteroid to use in this study in order to minimise the energy requirements and approximately meet our assumptions made in our equations. The masses of the most spherical asteroids such as Ceres are far too large, and since most small asteroids are non-spherical, the options were extremely limited. The best candidate was Mars' moon Phobos, which is not technically an asteroid, but a captured object that was likely an asteroid previously [2].

To begin with, we calculated the angular velocity required to make the centripetal acceleration experienced at the surface equal to 9.81 m/s^2. This required rearranging equation (1) for

ω and substituting $a_c = 9.81$ ms^{-2} and Phobos' mean radius $r = 11.276$ km [2]:

$$\omega_f = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9.81 \text{ ms}^{-2}}{11.276 \text{ km}}} = 2.95 \times 10^{-2} \text{ rads}^{-1} \quad (4)$$

We were then able to calculate Phobos' current angular velocity from its rotational period of $T = 7$h 39min, or $\Delta t = 27,540$ s.

$$\omega_i = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{27,540 \text{ s}} = 2.28 \times 10^{-4} \text{ rad s}^{-1} \quad (5)$$

Comparing the results of equations (4) and (5), the required increase in angular velocity is approximately two orders of magnitude.

The next step was to calculate the energy required to spin-up the asteroid from ω_i to ω_f. To do this, we took the expressions for the rotational energy associated with each of the angular velocities (equation (3)) and subtracted them, followed by substituting equation (2) for I.

$$\begin{aligned} \Delta E = E_f - E_i &= \frac{1}{2} I \left(\omega_f^2 - \omega_i^2\right) \\ &= \frac{1}{5} M r^2 \left(\omega_f^2 - \omega_i^2\right) \end{aligned} \quad (6)$$

We then calculated ΔE by substituting ω_f, ω_i, $r = 11.276$ km, and $M = 1.06 \times 10^{16}$ kg [2] into equation (6). The resulting value was $\Delta E = 2.35 \times 10^{20}$ J.

Discussion

This value for the required energy is very large and impractical to achieve in the modern era. For context, the combined energy output of all global nuclear weapons tests as of 1996 is 2.135×10^{18} J, which is only $\sim 1\%$ the energy required [3]. Moreover, with the most likely mechanism able to achieve this being large opposing rocket boosters, this amount of energy would require an enormous amount of propellant. Alternatively, some form of controlled and directed explosives could be deployed at antipodal points on the asteroid. This result means that if this concept was ever used, low mass asteroids would be a necessity given the energy requirements. However, this raises the concern as to whether the asteroid has the structural integrity necessary to survive rotating at a very high angular velocity. Small asteroids made of mostly loose rock and dust would be unlikely to remain in one piece.

It is important also to note that if an asteroid was successfully spun-up, only the equator of the asteroid would experience the Earth-like acceleration outwards, with structures at higher latitudes experiencing diminishing centripetal acceleration. Perhaps for the intention of mining an asteroid, an equatorial band around the asteroid would be sufficient for its inhabitants.

Conclusion

Our goal was to determine the feasibility of increasing the angular velocity of an asteroid to allow structures on the surface to have an artificial gravity strong enough to equal that felt on Earth. We found that for an asteroid similar to Phobos, the energy required to sufficiently increase its angular velocity would be of the order 10^{20} J. This is considerably larger than any value that could be harnessed by humanity today. As a result, we conclude that this scenario is unfeasible with today's technology. However, perhaps in the future, new ways of generating large amounts of energy could enable human explorers to make homes of asteroids using this method.

References

[1] P. Tipler and G. Mosca, Physics for Scientists and Engineers, (W.H. Freeman and Company, New York, 2008), 6th ed. (Academic, New York, 1980). Chapter 9-1.

[2] `https://iopscience.iop.org/article/10.1088/0004-637X/777/2/127/meta` [Accessed 28th October 2020]

[3] `https://books.google.co.uk/books?id=xAwAAAAAMBAJ&pg=PA63&dq=510.3&as_pt=MAGAZINES&ei=0VpnSuGLLoWQNuSdnaAB&redir_esc=y#v=onepage&q=510.3&f=false` [Accessed 28th October 2020]