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P3_3 30 Seconds To Mars

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Abstract

The aim of this paper is to calculate the velocity and total forces on an elevator climber such that it is able to reach the surface of Mars from Earth in 30 seconds. We calculated that the total length of such a machine would be $7.15 \cdot 10^{12}$ m, this is based on a $5.46 \cdot 10^{10}$ m (0.36 AU) elevator which is believed to be the shortest distance to Mars. The average velocity the climber required was found to be $6.0\dot{6}c$ and therefore physically impossible, despite this the forces acting on the climber should it have been possible were found to be $3.03 \cdot 10^9$ N

Introduction

As space travel grows increasingly closer with the success of both public and private projects, it is worth considering the time taken to reach destinations such as Mars. Using the well-developed idea of a space elevator, the stages a climber would go through can be decided and then the forces at each stage can be calculated. The aim of this paper is to look at the speeds required and the forces at each stage to reach Mars in the time frame suggested by American band '30 Seconds to Mars'. This already sounds like an ambitious time frame and it is worth noting this paper is purely hypothetical.

Theory

A paper published [1] in 2006 is used to find the equations needed to calculate the length of a space elevator and to account for many of the physical limitations the elevator would encounter, that are not mentioned in this paper. The 2006 paper states that the length can be calculated using:

$$H = \frac{R}{2}[(1 + 8(\frac{R_m}{R})^3)^{1/2} - 1] \tag{1}$$

where H is the elevator length in m, R_m represents the shortest distance to Mars and is equal to $5.46 \cdot 10^{10}$ m [2] and R is the radius of the Earth in m [1].

To calculate the velocity of the climber a simple equation of motion is used:

$$v = \frac{R_m}{t} \tag{2}$$

where v is the velocity of the climber in ms^{-1} and t is the time and equal to 30 s.

Given this will calculate an average velocity the journey can be split into two parts; the exit of Earth's atmosphere/gravitational pull and the low resistance space travel to mars. This can be seen as the parts where the lift cables pull the climber and where rockets propel the climber respectively. It is important to note that there is no consideration of a deceleration when arriving at Mars as this would require an instantaneous impulse or a higher average velocity during the travel, both of which would only create a more impossible model for this scenario. The two sections will be split up based on the fraction of the distance they account for. Part 1:

$$P_1 = \frac{Atm}{R_m} \tag{3}$$

where P_1 is part 1 of the distance to Mars and Atm is the distance to the atmosphere and is equal to $1 \cdot 10^5$ m [3]. Part 2:

$$P_2 = 1 - P_1 \tag{4}$$

where P_2 is part of the distance to Mars. The force will be equal at all times however each stage has different methods of creating the driving force. The lift uses tension in the cables to move the climber and rockets burn fuel to create a thrust force. This means a total tension can be found and a force per rocket. Assuming no resistive forces, the force can be calculated as:

$$F = \frac{\frac{1}{2}mv^2}{R_m} \tag{5}$$

where F is the total force in N and m is the mass of the climber in Kg.

Results

To calculate the total length of the elevator we used the length of our shaft which we assumed to be $5.46 \cdot 10^{10}$ m, the shortest distance to Mars. Using equation (1), the total length is found to be $H = 7.15 \cdot 10^{12}$ m. Using equation (2) the average velocity is found to be $6.0\dot{6}$c ms^{-1}. Using equation (3) and equation (4) the parts can be split into the two fractions $P_1 = 0.00000183$ and $P_2 = 0.99999817$. Using equation (5) and an assumption of 100 kg mass, the total force is $3.03 \cdot 10^9$ N. Therefore part one is 5545 N and part two is $3.03 \cdot 10^9$ N (3.s.f). Since the only upward force in a lift is the tension in the cables, if a shaft with two cables is assumed the force in each cable is 2772.5 N. As there are no current space elevators to reference it is assumed that the climber has a rocket on each vertical side (4). This means the total force for part to can be divided by 4 to give a force per rocket. This means the total force per rocket is equal to $7.57 \cdot 10^8$ N.

Discussion

The main point to take from these calculations and assumptions is that it is not only physically impossible to travel to Mars in such a short time, but the forces would also greatly exceed anything that is currently humanly possible. The current record for a rocket force is 40 MN which is over 10 times less than required by each rocket [4]. This design seems implausible even with an extended time to reduce the required velocity, as there would need to be extensive calculations put in to work out where the elevator would extend to and from to avoid collisions in space, along with considerations of the forces that would occur during acceleration and deceleration as they would not be instantaneous in reality.

Conclusion

We have shown that a space elevator taking up payload to Mars would require a length of $7.15 \cdot 10^{12}$ m. Though this length seems to be reasonable on a solar scale, the velocity required is 6 times greater than the speed of light. Therefore it must be concluded that '30 seconds to Mars' is not possible.

References

[1] `https://pdfs.semanticscholar.org/d402/ba5f97884b7398ae2a1ff79136f9c1a03993.pdf` [Accessed 25 October 2020]

[2] `https://mars.nasa.gov/all-about-mars/night-sky/close-approach/` [Accessed 25 October 2020]

[3] `https://www.space.com/17683-earth-atmosphere.html` [Accessed 1 December 2020]

[4] `https://web.archive.org/web/20051221102301/http://history.msfc.nasa.gov/saturn_apollo/documents/First_Stage.pdf` [Accessed 1 December 2020]