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P3_2 Reflection Under Pressure

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Abstract

Within this paper we calculate whether it is possible to position a large mirror within space such that a force from the radiation pressure of the Sun, would be great enough to adjust the position of the Sun. We calculate that the mirror would need to be of magnitude 30 times the angular size of the Sun in order to overcome the Sun's moment of inertia. It is clear from a practical perspective that this would not be possible.

Introduction

Radiation pressure is the force exerted on a surface due to electromagnetic radiation [1]. Photons do not have mass, however they do have momentum, p, given by

$$p = \frac{h}{\lambda} \tag{1}$$

where h is Planck's constant and λ is wavelength of the photon. The radiation pressure is due to the exchange of momentum from the photon to the surface. If the surface in question is reflective, then the reflection of the photon will also contribute to the radiation pressure. The total pressure, P_{tot}, is given by twice the incident pressure:

$$P_{tot} = 2\frac{I_f}{c} \tag{2}$$

Where I_f is incident intensity and c is the speed of light in a vacuum [2]. Using this basic theory that a photon can apply a pressure to a reflective surface, we can calculate whether it is possible to use a large mirror, positioned between the Earth and the Sun, eclipsing the Sun from the point of view of Earth, to apply a force great enough to move the Sun from it's position within the solar system.

Theory

The intensity at a given distance from the Sun is the average solar power per unit area, given by

$$I_f = \frac{3.8 \cdot 10^{26}}{4\pi r^2} \tag{3}$$

where the radius, r, is 1.6 million km, which is the radius of closest approach of the Parker Solar Probe [3]. This gives a power output of $11.81 \cdot 10^6 Wm^{-2}$. The radiation pressure, P_{tot}, is given by equation (2). The pressure in this case is $0.0787 Nm^{-2}$. Knowing this, we can find the force from radiation simply by

$$F_R = P_{tot} A \tag{4}$$

where A is the area of the mirror. To find this, we must consider what would be a reasonable selection for the diameter of the mirror. Let us decide that in order to capture the desired amount of radiation from the sun, the mirror should effectively eclipse the sun when viewed from Earth. In other words, the angular size of the mirror when viewed from Earth, δ, must be equivalent to the angular size of the Sun viewed from the same place (0.0093 radians) [4]. The angular diameter is given by

$$\delta = \frac{d_{mirror}}{D} \tag{5}$$

where d_{mirror} is the diameter of the mirror, and D is the distance to it. We know the distance from Earth to the mirror is $1AU - 1.6 \cdot 10^9 m$ and the angular diameter is given above, so a simple rearrangement allows us to find the diameter of the mirror, which comes out as $1.379 \cdot 10^9 m$. The area of the mirror can now be calculated simply as the area of a circular disc, and we can then substitute this area value back into equation (4).

To see whether this force is sufficient to move the Sun, we must calculate the moment of inertia for the Sun; as the Sun is not a sphere of uniform density, we find it's moment of inertia factor to be 0.070, the Mass of the sun is given by $M_\odot$, and radius by $R_\odot$ [5]. Then the moment of inertia, $I_\odot$, is given by

$$I_\odot = 0.070 M_\odot R_\odot^2 \tag{6}$$

If the force is less than the moment of inertia for the Sun, then the force generated due to the radiation pressure is not great enough to move the Sun, i.e. the force from the radiation pressure must overcome the moment of inertia.

Results and Discussion

The area of the mirror is found to be $1.488 \cdot 10^{18} m^2$, which gives a $F_R = 1.171 \cdot 10^{17} N$. The moment of inertia of the Sun is found to be $6.75 \cdot 10^{46} kgm^2$, which is roughly thirty orders of magnitude greater than the force due to radiation pressure, so the Sun could not be viably moved by a mirror of this size, though if the mirror were considerably larger it may be possible.

Conclusion

Using a stationary mirror of a size large enough to eclipse the Sun, and assuming the mirror is at an angle to the sun of $0°$, it is not possible to move the Sun using it's own radiation pressure. By increasing the size of the mirror by a magnitude of 30, we have shown that the force becomes large enough to overcome the Sun's moment of inertia. However, it is important to realise that whilst it is possible theoretically, it may not be possible to construct and launch a mirror of that size into the Sun's orbit in practise.

References

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[4] `https://www.austincc.edu/jheath/Solar/Hand/Angsize/asize15.htm#:~:text=The%20sun%20has%20an%20apparent,true%20diameter%20of%20the%20Sun.` [Accessed 20 October 2020]

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