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A1_2 Seismic Superman

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Abstract

In this paper we introduce Superman's new power, Seismic Shock. When he is shot at, he transforms the bullets' energy into earthquakes which resonate with different building heights at different firing rates. It only takes thirty henchmen shooting to take down a high-rise building, while one and two storey buildings require roughly $200 - 800$ henchmen.

Introduction

Superman is famous for his extraordinary abilities, such as the ability to stop bullets, which raises the question of what he does with the energy he absorbs in the process. Therefore, we propose a new ability, the creation of seismic shocks from the bullets' energy. We analyse the rate at which he is fired at by multiple people, and the corresponding seismic waves with different frequencies that will resonate with differently sized buildings to collapse them. Here, we use simple mechanical wave models for the earthquakes and the buildings.

Theory

To be impervious to bullets, Superman must absorb their kinetic energy, as they hit him such that they stop and clatter to the floor. Each bullet with a mass m_b and velocity v_b carries kinetic energy $KE = m_b v_b^2/2$. If Superman is fired at at a rate of $R_b = N$ bullets (rounds) per second, then he absorbs power P_b

$$P_b = \frac{1}{2} R_b m_b v_b^2. \tag{1}$$

Here, we use the simplest possible mechanical analysis by modelling the earthquake as a planar transverse shear s-wave. Its generalized planar wave form is described by $\mathbf{A} = \mathbf{A_0} \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$ where the amplitude $\mathbf{A_0}$ is perpendicular to $\mathbf{k}$ for a transverse wave. The power P_E of the wave is given by

$$P_E = Z \left(\frac{\partial A}{\partial t} \right) = Z A_0^2 \omega^2 \cos^2(kr - \omega t) \tag{2}$$

where $Z = \rho_s v_s$ is the impedance of the ground, ρ_s is its density, and v_s is the velocity of the s-wave [1]. The average power $\langle P_E \rangle$ carried by the wave is

$$\langle P_E \rangle = \frac{1}{2} \rho_s v_s A_0^2 \omega^2. \tag{3}$$

Assuming all of the power Superman absorbs from the bullets is directly transferred into the earthquake, such that $P_b = \langle P_E \rangle$, solving for the frequency ω gives

$$\omega = \frac{v_b}{A_0} \left[\frac{R_b m_b}{\rho_s v_s} \right]^{1/2}. \tag{4}$$

To examine the response of the buildings to Superman's earthquake, we find their resonance frequencies. The resonant frequency is usually $\omega_0 = \sqrt{k/m}$ where k is the spring constant and m is the building mass. The spring constant of a wall is found using $k = ew/h$ where e is the elastic modulus of the material, w is the wall thickness, and h is the building height [2]. Similarly, we must derive the mass of the building as a function of height. We model all buildings as rectangular prisms with a constant floor area A and assume that 10% of this floor area is covered by walls where the wall volume is $V_w = 0.1Ah$. With a wall density of $\rho_c = m/V_w = m/0.1Ah$, the mass as a function of height is given by $m(h) = 0.1Ah\rho_c$. Therefore, the relationship between the resonant frequency and the height of the building can be expressed as

$$\omega_0 = \frac{1}{h}\left[\frac{10ew}{A\rho_c}\right]^{1/2}. \tag{5}$$

To find the height of the building at which its resonant frequency responds to the seismic wave frequency (dependent on the firing rate), we solve for $h = h(R_b)$ by equating $\omega = \omega_0$ (Eq. 4 and 5) and using constants $C_1 = v_b/A_0\sqrt{m_b/\rho_s v_s}$ in Eq. 4 and $C_2 = \sqrt{ew/A\rho_c}$ in Eq. 5 which gives

$$h = \frac{C_2}{C_1}R_b^{-1/2}. \tag{6}$$

Results

Fig. 1 shows the plot of Eq. 6, $h(R_b)$ with the following constants. The submachine gun specifications are a muzzle velocity of $v_b = 700\,\mathrm{m\,s^{-1}}$ and bullets weighing $m_b = 2\,\mathrm{g}$ [3]. We also model the ground as sandstone, with $\rho_s = 2600\,\mathrm{kg\,m^{-3}}$ and $v_s = 2800\,\mathrm{m\,s^{-1}}$ [4, 5]. Finally, we assume a floor area $A = 90\,\mathrm{m}^2$ and wall thickness $w = 0.3\,\mathrm{m}$ for the buildings, which are made out of concrete with $\rho_c = 2300\,\mathrm{kg\,m^{-3}}$ and $e = 30\,\mathrm{GPa}$ [6]. One storey is about 5 m.

Conclusion

The inverse relationship between the height of a resonating building that will resonate and the rate of bullets is because lower frequency waves – with greater wavelengths, or h – contain less energy. To collapse a high rise building (Fig. 1), Superman has to be hit at rate of ~ 600 rps. A submachine gun can fire at ~ 20 rps [3], so 30 people will need to shoot him simultaneously in Metropolis. In the suburbs, he must fight against at least $200-700$ people to bring the buildings to fall, and at home on his farm that reaches > 800 people for one-storey buildings. This is not a thorough analysis of the earthquakes Superman creates, and buildings resonate at different values of R_b since an earthquake is made of multiple waves with different typical frequencies, velocities, and wave forms. In conclusion, it is more energy efficient for Superman to fight in the city rather than the suburbs or country-side.

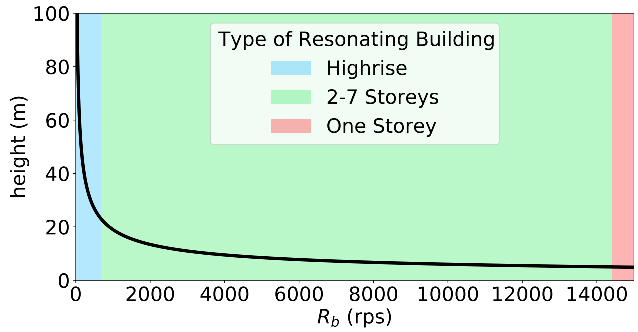

Figure 1: The relationship between the firing rate and the height of the building that will resonate with the corresponding earthquake (Eq. 6).

References

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