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P4_02 Lorentz Symmetry Breaking through a hypothetical Compton Effect

Rishan Dev Choudhury, Robert Savage, Thomas Rawlins and Adam Hammerton
School of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract
Lorentz Invariance is upheld by the two postulates of special theory of relativity. This paper considers the Compton effect at very high energies and by using the Compton Effect Scattering equations and proposing the speed of light to be a function of frequency, two new events were worked out. These being: no scattered photon and a scattered photon with higher energy. These situations can be considered as verifying signatures for Lorentz Invariance Violation (LIV) in high energy regions of the Universe.

Introduction

The speed of light in vacuum is constant which is one of the postulates of special relativity. Instead of speeding up, when a photon becomes more energetic, it's frequency (ν) increases. Presently, a lot of research is undergoing to study that light may have different speeds in a very-high energy regime. [1] This is termed as Lorentz Invariance Violation (LIV). Since, LIV is upheld in high energy regime, and the only physical characteristic that implies the different energy of the photon is frequency, we assume $c = f(\nu)$.

Theory

Just like the resistance (R) dependence on temperature is written as $R_t = R_0(1+\alpha t_1+\beta t_2+....)$, we consider

$$c^{'} = f(\nu) \equiv c_0[1 + \alpha(\nu^{'} - \nu_0) + ...] \qquad (1)$$

where $c_0 \equiv c$ = usual speed of light of a photon having the frequency ν_0 (in vacuum); $c^{'}$ = speed of the photon of frequency $\nu^{'}$ in vacuum.

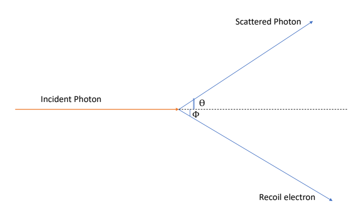

Figure 1: Compton Scattering [Made on Photoshop]

Again from (1), we have $c^{'} = c_0[1 + \alpha\Delta\nu]$, keeping only the first order term and $\alpha = \frac{c^{'} - c_0}{c_0 \Delta\nu}$, where $\nu^{'} - \nu_0 = \Delta\nu$. If $\Delta\nu << 1$, and $\alpha \to 0$, then $c^{'} = c_0$ thus implying that Lorentz Symmetry is still obtained.

In our proposed hypothetical Compton Effect:
(i) Since the incident photon and scattered photon are different, we assign them different

speeds say c_0($\equiv$ usual speed of light) and $c^{'}$.
(ii) Following the same procedure in usual Compton effect [2] i.e. the principles of Conservation of Energy and Momentum, we obtain the following relation:

$$h\nu + m_0c^2 = h\nu^{'} + mc^2 \tag{2}$$

where m_0 and m are the rest mass of the electron and relativistic mass of the recoiled electron respectively. Again, taking the component forms of (2) and using Fig (1), we obtain

$$\frac{h\nu}{c_0} + 0 = \frac{h\nu^{'}}{c_0(1+\alpha\Delta\nu)}\cos\theta + mv\cos\phi \tag{3}$$

$$0 = \frac{h\nu^{'}}{c_0(1+\alpha\Delta\nu)}\sin\theta - mv\sin\phi \tag{4}$$

Now, taking the usual follow-up of normal Compton effect, we ultimately obtain

$$\begin{aligned} -2h\left(\nu-\nu^{'}\right)m_0c^2 &= -2h^2\nu\nu^{'}\left[1-\frac{\cos\theta}{1+\alpha\Delta\nu}\right] \\ &\quad - h^2\nu^{'2}\left[1-\frac{1}{(1+\alpha\Delta\nu)^2}\right] \\ \Rightarrow \frac{1}{\nu^{'}} - \frac{1}{\nu} &= \frac{h}{m_0c^2}\left[1-\frac{\cos\theta}{1+\alpha\Delta\nu}\right] \\ &\quad - \frac{h\nu^{'}}{2m_0c^2\nu}\left[1-\frac{1}{(1+\alpha\Delta\nu)^2}\right] \end{aligned} \tag{5}$$

Result and Discussion

(a) If $\alpha\Delta\nu \approx 0$ then we have from (5)

$$\frac{1}{\nu^{'}} - \frac{1}{\nu} = \frac{h}{m_0c^2}\left[1-\cos\theta\right] \tag{6}$$

which is the usual Compton Effect at low-energy regimes.

(b) Let there be no scattering i.e. $\theta \cong 0$; so $\nu = \nu^{'}$. Then from (5),

$$\Rightarrow \nu^{'} = \frac{2\nu}{1+\alpha\Delta\nu} \tag{7}$$

This condition can be imagined only for a highly energetic photon. Even if $\nu^{'} \neq \nu$, we see that $\frac{1}{\nu^{'}} = \frac{1}{\nu} \cong 0$ since the frequency tends to infinity in very-high energy regime.

(c) Let the condition be considered where the scattered photon becomes more energetic. So from (5), we have

$$\begin{aligned} &\frac{h\nu^{'}}{2m_0c^2\nu}\left[1-\frac{1}{(1+\alpha\Delta\nu)^2}\right] > \frac{h}{m_0c^2}\left[1-\frac{\cos\theta}{1+\alpha\Delta\nu}\right] \\ &\Rightarrow \nu^{'} > 2\nu\frac{\left[1-\frac{\cos\theta}{1+\alpha\Delta\nu}\right]}{\left[1-\frac{1}{(1+\alpha\Delta\nu)^2}\right]} \approx 2\nu\frac{\left[1-\frac{\cos\theta}{1+\alpha\Delta\nu}\right]}{\frac{2\alpha\Delta\nu+\alpha^2(\Delta\nu)^2}{(1+\alpha\Delta\nu)^2}} \\ &\Rightarrow \nu^{'} > \nu\left(1+\alpha\Delta\nu\right)\left[1+\frac{2\sin^2\frac{\theta}{2}}{\alpha\Delta\nu}\right] \end{aligned} \tag{8}$$

Thus, after being scattered on a hypothetical object, the photon becomes more energetic. In other words, it is blue-shifted. Physically, this may be imagined in the neighbourhood of a blackhole [3] where the photon's acceleration towards a blackhole is not negligible.

Conclusion

Through our calculations we found three unique events, the first of which being in the low energy region where the speed of light is constant and the usual Compton Scattering is obtained. The second and third events show no scattered photon and a photon receiving higher energy than the incident one, these events are calculated to occur in high energy regions and therefore Lorentz Invariance Violation happens in the high energy regime.

References

[1] Tipler and Mosca, *Physics for Scientist and Engineers*, 6th Edition, Compton Scattering (Pg 1178)

[2] Stefano Liberati, *Lorentz Symmetry breaking: phenomenology and constraints*, J. Phys.: Conf. Ser. **631** 012011(2015)

[3] D. Simpson, *A Mathematical Derivation of the General Relativistic Schwarchild Metric*, B.Sc. Thesis, ETS University (2007)