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P3_8 Narnia and the Theory of Special Relativity

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Abstract

We investigated the velocities Narnia would have to be moving at relative to Earth for the different lengths of time that pass in Narnia compared to on Earth in 'The Chronicles of Narnia' books. The relative velocities that were calculated at different points when the characters visited it ranged from $0.986013297c$ to $0.999999700c$, where c is the speed of light. We then calculated the corresponding distances of Narnia from Earth, which were found to range from $\sim$4270 Mpc to $\sim$4330 Mpc.

Introduction

The fictional world 'Narnia', is visited several times during the book series 'The Chronicles of Narnia'. Time on Earth appears to run slower than in Narnia, with the time from Narnia's creation to its end spanning 2555 years in Narnia, but only 49 years on Earth [1]. Using Einstein's theory of special relativity, we find the velocities Narnia must be moving at relative to Earth for different amounts of time to have passed in both places, if it was a real place on a planet in another galaxy. This theory explains why time runs slower in a reference frame moving at a high relative velocity to another reference frame, an effect known as time dilation. The effects of this are more obvious at speeds close to light speed [2]. We also use Hubble's constant to find the distances Narnia would have to be from Earth for it to be moving away with these velocities.

Theory and Results

From the first visit in 1900 to when Peter, Susan, Lucy and Edmund Pevensie go there in 1940 (Earth time), 1000 years have passed in Narnia [1].

We used the time dilation equation to find the velocity that Narnia would have to be travelling at in the rest frame of Earth:

$$v = c\sqrt{1 - \frac{t^2}{t'^2}}, \qquad (1)$$

where t' is the dilated time in Narnia, t is the proper time on Earth, c is the speed of light (299792.458 km s^{-1}) and v is the velocity of Narnia relative to Earth. The relative velocity of Earth to Narnia is the same but in the opposite direction, so Earth's time appears to run slower observed from Narnia.

Therefore the realtive velocity of Narnia at point of the trip in 1940 is $0.999199680c$.

As the universe is expanding, Narnia's galaxy will move away from the Earth in a radial direction. Therefore the distance of Narnia from Earth, if it is moving at this velocity can be found using the equation:

$$d = \frac{v}{H_0}, \qquad (2)$$

where d is the distance of Narnia from Earth, v is the relative velocity of Narnia's galaxy in a radial direction away from Earth and H_0 is the Hubble constant (69.3 km s^{-1} Mpc^{-1}), which is the current rate of expansion of the universe [3]. This means the distance of Narnia from Earth would be ~4320 Mpc in 1940. 1 Mpc is equal to 3.086×10^{19} km.

After returning to England, the Pevensies visit Narnia again after 1 year on Earth but 1288 years have passed there since they left, so the relative speed at this point is 0.999999700c, with its distance from Earth being ~4330 Mpc.

In summer 1942, Lucy and Edmund travel to Narnia again with their cousin, Eustace. They find that it has only been 3 years in Narnia since the last trip and we assume that about half a year has passed on Earth. The relative velocity of Narnia and its distance from Earth at this point are calculated to be 0.986013297c and ~4270 Mpc respectively.

We assume that the following trip to Narnia in autumn 1942 takes place ~0.25 years after this. It has been 49 years in Narnia since Lucy, Edmund and Eustace left, so its relative speed is 0.999986984c and the distance is ~4330 Mpc.

199 years later in Narnia, in 1949 on Earth, ~6.25 years after the trip in 1942, the last visit to Narnia takes place during the apocalypse of the fictional world. Thus, the relative speed at this point would be 0.999506679c, with a distance from Earth of ~4320 Mpc.

Discussion

In the books, the characters are transported instantly between the two worlds, so they would be travelling faster than the speed of light. This is impossible because any object with mass travelling at light speed would have infinite energy [4]. This instant travel would appear to be time travel in some frames of reference and would thus break causality.

The times in both worlds when Narnia is visited have only been determined to the nearest year or season from source [1], resulting in an error in our results. The value used for Hubble's constant is found from the combination of cosmological data and WMAP data which combines different types of measurements and has an uncertainty of 1%, increasing the uncertainty in the values for the distance.

As the time in Narnia is not directly proportional to Earth's time, the relative velocity of Narnia in the rest frame of Earth increases and decreases randomly, but we assume this velocity is constant when the characters travel from one world to the other. The time dilation equation shows that the dilated time is independent of the acceleration and only depends on the current velocity, according to the clock postulate [5]. The variations in the results suggest Narnia must move towards and away from Earth at different times to allow it reach different distances, so its motion is not only due to the expansion of the universe, which could lead to errors in our values for distance, which are calculated using the assumption that Narnia's relative velocity is only due to universal expansion.

Conclusion

We calculated the relative velocities of Narnia to Earth at different points from the first to the last trip to Narnia, as well as the corresponding distances from Earth. The variations in our results suggest that Narnia's galaxy would have to be moving towards Earth at some points, so its relative motion is not only a result of the expansion of the universe. This could be partly due to errors in the results.

References

[1] W. Hooper, *Past Watchful Dragons: The Narnian Chronicles of C. S. Lewis* (Macmillan Publishers, New York, 1979).

[2] P. A. Tipler and G. P. Mosca, *Physics for Scientists and Engineers: with Modern Physics* (W. H. Freeman and Co., New York, 2008), 6^{th} ed., chapters R and 39.

[3] `https://tinyurl.com/wtkvwu8` [Accessed 26 November 2019]

[4] `https://tinyurl.com/resaltl` [Accessed 26 November 2019]

[5] `https://tinyurl.com/qbc6prr` [Accessed 26 November 2019]