A4_13 Dr. Evil's "Laser"

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Abstract

In the film Austin Powers: The Spy Who Shagged Me, Dr. Evil threatens to blow up Washington, D.C. by using a laser located on the Moon. In this paper, we investigate the energy needed to blow up the Earth with this type of laser and explore the characteristics a laser like this would have. We also comment of the feasibility of the laser. We have discovered that this laser would have to have an energy of 2.26×10^{32} J and is not even theoretically possible.

Introduction

Austin Powers: The Spy Who Shagged Me is the second film in the Austin Powers comedy trilogy. In the film, the main antagonist, Dr. Evil, attempts to blackmail the US government by threatening to blow up Washington, D.C. with a laser situated on the Moon. In this paper, we discover the energy needed to blow up the Earth from a laser on the Moon and explore the various technical characteristics a laser of this kind would have. Throughout this paper we will ignore the heating and focal effects upon the laser equipment.

Theory

To discover the energy needed to blow up the Earth, and thus the required laser energy, we used the gravitational binding energy. The gravitational binding energy is the amount of energy needed for a system to lose its gravitational bound state. We assume the Earth is a spherical mass of uniform density. We can find the gravitational binding energy of the Earth (U) using Equation (1) from source [1]. In this equation, M is the mass of the Earth, G is the gravitational

constant, and R is radius of the Earth.

$$U = \frac{3GM^2}{5R} \quad (1)$$

Due to the nature of propagating waves, a laser will always diverge when in a homogeneous medium. However, for the sake of simplicity we will assume that the laser is a highly collimated Gaussian beam. As such the beam divergence of a Gaussian beam is calculated using Equation (2) from source [2]. In this equation, from the source, there should be a M^2 (beam quality) factor. As we are making the assumption of a perfect beam, this factor equals one. θ is the radial beam divergence which is equal to the angular beam divergence (Θ) divided by 2.

$$\Theta = 2\theta = \frac{\lambda}{\pi w} \quad (2)$$

In this equation, w is the radius of the beam at its narrowest point, known as the "beam waist" and λ is the wavelength of the laser, which we calculate using Equation (3).

$$U = E = \frac{hc}{\lambda} \quad (3)$$

Knowing Θ, we are also able to calculate the diameter of the laser beam at Earth, which we

do using a rearranged form of Equation (4). In this equation, D_f and D_i are the beam diameters at the surface of the Earth and the Moon respectively, and l is the distance from the Moon to the Earth.

$$\Theta = 2\arctan\left(\frac{D_f - D_i}{2l}\right) \qquad (4)$$

Once we have found the value of D_f, we can use this to calculate the energy density of the laser, otherwise known as the "fluency" of the laser on this area, using Equation (5). Energy density is the most important value as it shows how much energy will be concentrated to a certain area. If the laser beam covers a large area due to the divergence, then the energy will be "spread" out and the Earth will not explode.

$$Fluency = \frac{U}{\pi \left(\frac{D_f}{2}\right)^2} \qquad (5)$$

Results

Given that the mass and radius of Earth are 6×10^{24} kg and 6371 km respectively [3], the gravitational binding energy of Earth using Equation (1) is 2.26×10^{32} J. Meaning that a laser would have to produce this amount of energy to blow up the Earth. Using Equation (3), this equates to a wavelength of 8.79×10^{-58} m. From this, we use λ to calculate the beam divergence of our Gaussian beam, as stated in Equation (2). We assume that this laser will be very highly technologically advanced due to its location on the Moon and energy capability. The ELI-NP laser in Romania is the most powerful laser in the world to date [4]. It has an initial beam diameter of 450 mm [4] and thus a beam waist of $w = 225$ mm. We will use this value for our laser and plug into Equation (2). From this, the beam divergence, Θ is calculated to be $2.49 \times 10^{-57\circ}$. The average distance between the Earth and the Moon is $384,400$ km [5]. If we rearrange Equation (4) to calculate D_f and substitute $l = 384,400$ km, we find that the beam diameter of the laser at Earth will be 450 mm. The divergence of the beam is negligible and so even over such a large distance, it does not affect the beam diameter at Earth. Because the spot size of the laser on the Earth's surface is so small, the curvature of Earth does not have an affect. We also ignore the effects of light scattering from the atmosphere. Knowing this value we can achieve an energy density of 1.42×10^{33} J m^{-2} on the Earth from the laser.

Discussion

As a comparison to current technology, the smallest wavelength achieved by a laser is 1.5 Å, which is an atomic X-ray laser [6]. In 2019, the highest-energy gamma rays were observed from the Crab nebula with 7.21×10^{-5} J photons which correlate to a wavelength of 2.76×10^{-21} m [7]. The wavelength of the laser is beyond anything we know today. It is smaller than the Planck length, meaning that it would be theoretically impossible for this to occur. Even if one discretises a quantum field, the coarse grid would have the scaling of the Planck length. This wavelength would therefore create an impossible situation, as one can tell from the Heisenberg uncertainty principle. If a laser were capable of exploding the Earth we would see a host of interesting phenomena. The atmosphere would instantaneously turn to plasma. Nuclear interactions such as fusion and pair production would also occur turning the Earth into a "fireball".

Conclusion

To conclude, we have discovered the energy needed for a laser to explode the Earth from the Moon is 2.26×10^{32} J. We then proceeded to investigate the properties this kind of laser would have and comment on the feasibility (or lack of) of creating such a laser with today's technology.

References

[1] *Astrophysical Formulae*, K.R.Lang (1980), Springer Verlag, p.272

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[4] *"Where does ELI-NP stand now?"– Phase II*, Kazuo A. Tanaka (2018), ELI-NP

[5] `https://bit.ly/351eypg`[Accessed 26/11/19]

[6] `https://bit.ly/2KZOcw6`[Accessed 26/11/19]

[7] `https://bit.ly/2DRDynf`[Accessed 26/11/19]