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A4_12 Beyond the scope of this paper

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Abstract

A telescope made from pages from the Journal of Physics Special Topics was modelled in this paper. It was determined that a user of this telescope would theoretically be able to read a piece of writing from this journal at a distance of 9.3 cm. However, when scattering effects and the assumption of this paper are considered/removed, it would become implausible for this to be used for real-world applications.

Introduction

In this paper, we determine the distance that a person can read a paper from the Journal of Physics Special Topics using a telescope entirely made from paper from said journal (including the lenses). We define the readable distance as being able to resolve the spacing between letters of text.

Theory

We must first determine the magnification of a two lens telescope, which is shown in equation (1). In this equation M is the magnification, f_o is the focal length of the *objective* lens and f_e is the focal length of the *eyepiece* lens.

$$M = -\frac{f_o}{f_e} \quad [1] \tag{1}$$

We can determine the focal length of the paper by using the radius of curvature. If one rolled an A4 piece of paper (dimensions being 210 $\times$ 297 mm [2]) to make a tube, then the diameter of said tube would be 66.8 mm. For a lens to snugly fit in this pipe, the convex radius of curvature would therefore be 33.4 mm (assuming a perfectly circular shape with no creases). As 80 gsm A4 bond paper, a common paper for printing, has a thickness of 0.1 mm, the concave radius of curvature would be 33.3 mm. We can then use the lens-maker equation to determine the focal length of the lenses. In this equation f is used as this can be either the objective or eyepiece focal length, n is the refractive index of the paper, n_{air} is the refractive index of air (which will be assumed to be 1), and r_1 and r_2 are the radii of curvature for the lens. For clarity a diagram of this model is shown in figure (1).

$$f = \frac{1}{\left(\frac{n}{n_{air}} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \quad [4] \tag{2}$$

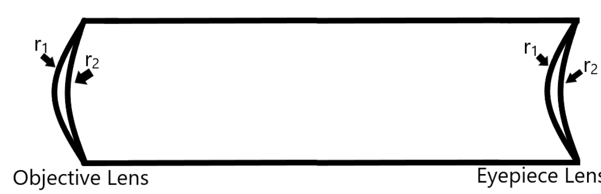

Figure 1: Diagram showing the telescope

The refractive index of paper is 1.55 with the coating particles having indices varying from 1.5-2.7 [5]. We assume that the eye-piece lens has the higher refractive index from this range (by assuming the paper has a large ink coverage) and the objective lens has the refractive index of standard paper. These values can then be substituted into equation (2) to get the focal lengths, which can then be substituted into equation (1) to get a magnification.

The human eye has a maximum angular resolution of 1 arcminute (0.02°) [6]. The guidelines for the Journal of Physics Special Topics have a line spacing of 0.6 mm. By using the angular magnification of our telescope we can determine the maximum distance at which this letter spacing can be resolved. This is shown in equation (3), where we have used Pythagoras's formula and the small angle approximation ($\sin(\theta) \approx \theta$). In equation (3): L is the maximum distance, d is the letter spacing and θ is the angular resolution of the eye.

$$L = \left| \frac{Md}{\theta} \right| \tag{3}$$

The final formula for this maximum length away from the paper is shown in equation (4), where n_e is the refractive index of the eyepiece paper and n_o is the refractive index of the objective lens paper. In this formula the radii of curvature have cancelled due to both lenses having the same values.

$$L = \frac{(n_e - 1)\, d}{(n_o - 1)\, \theta} \tag{4}$$

Assumptions

The largest assumption in this paper is that scattering effects are not occurring. For standard 80 gsm bond paper, this would not be the case. We have assumed it is for this paper, as it allows up to calculate the theoretically achievable magnification. When the paper is bent to create a lens, we also assumed it would be able to be perfectly circular with no creases. As a flat piece of paper would have a Gaussian curvature of zero initially, this would not be possible, as this needs to be conserved in the lens making process. We have assumed this, however, to vastly simplify the optical equations that are required.

Results & Discussion

Substituting in our values into equation (4) gives a value of 9.3 cm. This is 15.7 cm shorter than the near point for a human eye [7]. This means that the telescope, if only this is considered, would not be completely pointless, and would work well as a pseudo-magnifying glass. However, the scattering of light that occurs due to the paper in the visible spectrum is very significant, meaning that one would not be able to see the magnified image [8].

Conclusion

We have determined that the magnification of a telescope made from a paper from the Journal of Physics Special Topics would theoretically allow a user to read text at a distance of 9.3 cm. However, in reality the scattering effects that occur due to the paper would cause the text to be unreadable. Future studies could quantify this scattering to determine what the *actual* readable distance would be, as this was... beyond the scope of this paper.

References

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