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A6_8 The Hyperloop through the centre of the Earth II

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Abstract

In previous research it was shown how a 500 kg Hyperloop pod falling through the centre of the Earth would never return to the surface when air resistance is considered. In this paper two possible methods to ensure that the pod would reach the opposite side of the Earth are considered; a thruster and a winch. For 5000 kg of fuel, it was determined that the total travel time using thrusters would be 2820 s if all the fuel is burnt instantaneously, making it a feasible solution. On the other hand, the travel time using a winch was over 86 hours. Therefore using such a method would not be particularly useful given assumptions made here.

Introduction

In the first paper [1], differential equations describing the motion of a 500 kg Hyperloop 'pod' as it travels in a tube through the centre of the Earth were derived. The motion of the pod when air resistance in the tube was, and was not negligible was examined. In the case of the latter it was shown that the motion was described by

$$\frac{d^2y}{dt^2} + \frac{\rho_{air} C_D A}{2m_{pod}} \left(\frac{dy}{dt}\right)^2 - \frac{4}{3} G \rho_{earth} \pi y = 0, \quad (1)$$

where y is the position of the pod, m_{pod} is the mass of the pod and C_D is the drag coefficient. All other terms have their usual meaning and are defined in [1]. The numerical solution to (1) showed the amplitude of the pod's oscillation decaying exponentially with time, therefore the pod could not be used as a method of transport.

In this paper, two modifications to the Hyperloop system are considered such that the pod would reach the opposite side of the Earth. The first method uses thrusters and the second considers a winch.

Method

The possibility of using thrusters to 'boost' the pod towards the opposite side of the Earth were examined first. The mass of fuel required was assumed to be 5000 kg (this number is arbitrary and can be adjusted for a more specific case), such that the total mass at launch is $m_T = 5500$ kg. To provide a velocity change, Δv, all fuel will be burnt instantaneously at the position the pod changes it's direction for the first time - i.e. as it begins moving back towards the centre of the Earth. Setting $m_{pod} = m_T$ and applying the same initial conditions as before, the time (t_t) and position (y_t) of the first direction change were found by identifying the co-ordinates of the first turning point in the solution to (1). After the burn, the pod would enter a new phase of motion described by the solution to (1) with new initial conditions; $y = y_t$, $t = t_t$, $v = \Delta v$ and m_{pod} = 500 kg. A range of values for Δv were used in a graphical trial and improvement method to determine the velocity change required to return the pod the opposite side of the Earth, $y = (\text{-})\ R_E$ (but no further). The suitable value of Δv was then used to determine the specific impulse of the boost given by

$$I_{sp} = \frac{\Delta v}{g_0 \ln(m_0/m_f)}, \tag{2}$$

where g_0 is the surface gravity and m_0 and m_f are the initial and final masses respectively. The specific impulse can be used to determine the efficiency of the burn.

An alternative method of bringing the pod to the opposite side of the Earth would be to 'catch' the pod in a net at the centre of the Earth. A winch would then be used to bring the pod the surface. The energy required to move the pod from the centre to the surface can be calculated by integrating the gravitational potential energy over the radius of the Earth. The final result is

$$E = \frac{2}{3}\pi G m_{pod} \rho_E R_E^2, \tag{3}$$

where all symbols have their usual meaning.

Results and Analysis

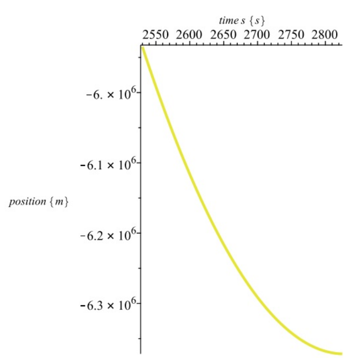

Figure 1: Solution to (1) for the second phase of motion; Δv = 2930 ms^{-1}. The origin here represents the co-ordinates of the turning point found in the first phase.

For the thruster method, the pod first changed direction at y_t = - 5.94×10^6 m, corresponding to $t_t \approx 2530$ s. The appropriate value of Δv was found to be approximately 2930 ms^{-1}. In reality, the velocity change would be applied over a short period of time. From Figure 1, the travel time for the second phase (after the boost) was found to be 270 s, and a total travel time of 2820 s. The required specific impulse was calculated as 124 s which is noticeably smaller than that of a Saturn V rocket - 260 s [2]. This implies creating a nozzle of this efficiency would be feasible.

In the case of the winch, the energy required to bring the pod to the surface was found to be ≈ 284 kJ. The time taken to travel a distance 1 R_E was found to be ≈ 86 hours using the relationship between speed, distance and time. The velocity of the pod was assumed to be the same as that of the worlds' fastest elevator (Nex-Way, China) - 73.8 kmh^{-1} [3]. The definition of power was used with the required energy and calculated travel time to show the average power required to lift the elevator is ≈ 9 W.

Conclusion

In this paper it was shown that the time of travel using thrusters is faster than conventional air travel, whereas the use of a winch is not. A faster lift velocity could be used, however thrusters may be needed to provide this anyway. With the travel time and specific impulse calculations it can be concluded that the use of thrusters is a feasible solution to the problem. In the future, a consideration of different masses of fuel could be made, or the position of the velocity change could be varied.

References

[1] M. Favaro-Bedford, R. Lindon, E. Day, C. Wallis, J. Mayes, *A6_4 The Hyperloop through the centre of the Earth*, PST 17, (2018)

[2] `https://history.msfc.nasa.gov/saturn_apollo/documents/F-1_Engine.pdf`[Accessed 27 November 2018]

[3] `http://www.guinnessworldrecords.com/world-records/fastest-lift-(elevator)/`[Accessed 27 November 2018]