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P5_6 Compton Scattering Shapes

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Abstract

In this paper we discuss how the wavelength of a photon would change if repeatedly scattered, via the Compton scattering process, by stationary electrons aligned such that the photon traces out a regular polygon of n sides. We find that for small n there is a much larger change in wavelength which rapidly decreases, then the change in wavelength asymptotically approaches 0.

Introduction

Compton scattering is a process describing the scattering of a photon by a charged particle, most commonly an electron. In this process the photon changes direction, losing energy in the process, hence, in accordance to equation 1 [1], changes wavelength.

$$\Delta E = \frac{hc}{\Delta \lambda} \tag{1}$$

This scattering can happen consecutively, multiple times. This means if there are electrons arranged in an appropriate configuration a photon can scatter multiple times such that it returns to some initial point, thus tracing out the perimeter of a shape. In this paper we explore what the change in wavelength will be when the angles a photon scatters through are such that they trace out a regular polygon of n sides (see Figure 1).

Theory

The change in wavelength due to Compton scattering is given by equation 2 [1],

$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{mc}\left(1 - \cos\theta\right) \tag{2}$$

where h is the Planck constant, c is the speed of light, m is the mass of an electron and θ is the angle the photon is scattered through.

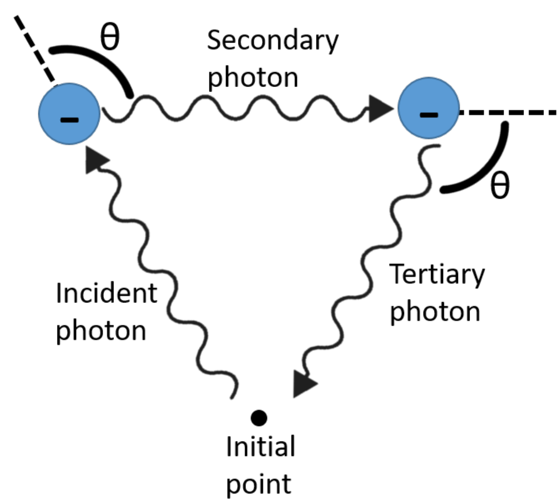

Figure 1: A photon being scattered such that it traces a triangle ($n = 3$). The electrons are shown in blue. Note that once the electrons have interacted with the photon, they too would scatter. To maintain simplicity this has not been shown in this figure.

For the photon to be scattered such that it traces the path of a regular polygon with n sides, we consider a system consisting of $n-1$ stationary electrons, and pointing to the vertices of the polygon. To meet our condition, the photon will be scattered through an angle equal to the exterior angle of the polygon $n-1$ times. This is shown for $n = 3$ in Figure 1.

The exterior angles of a regular polygon with n sides is simply given by $2\pi/n$ and to trace out the polygon the scattering process must be repeated $n-1$ times, thus the consequent total change in wavelength is given by equation 3.

$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{mc}(n-1)\left(1 - \cos\left(\frac{2\pi}{n}\right)\right) \quad (3)$$

Results

Plotting equation 3 for the first 98 polygons (i.e. $n = 3$ to $n = 100$) we obtain the plot shown in Figure 2.

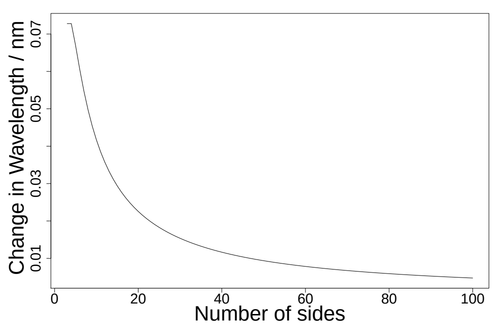

Figure 2: Rplot showing how the change in wavelength of a photon, scatting such that it traces out a polygon with n sides, relates to the number of sides of the corresponding polygon.

Discussion

Looking at Figure 2 we can see that for the polygons with fewer sides, the change in wavelength is greater. This is because the exterior angles of polygons with fewer sides is greater. Thus, the photon is scattered over a greater angle, thus it loses more energy, hence there is a greater change in wavelength.

From this we learn that, given the condition that we want scattering photons to trace out regular polygons, the size of the angle the photon is scattered through is more significant in changing the photon's wavelength that the number of times the photon is scattered.

It is worth noting that the change in wavelength for a photon scattered such that it traces out a triangle, $n = 3$, is the same as that which traces out a square, $n = 4$. This is a special case where despite having smaller scattering angles, the extra scattering event for $n = 4$ results in sufficient energy loss to equate that of $n = 3$ resulting in the same change in wavelength.

As n increases the energy loss due to the angle the photon scatters through becomes less significant due to the exterior angles becoming smaller. Also, the rate at which the change in wavelength decreases for successive values of n decreases due to the difference between the exterior angles of successive decreasing.

The graph asymptotically approaches a change in wavelength of 0, since as n approaches infinity the scattering angle, $2\pi/n$, approaches 0. Physically, this means though the angles the photons are being scattered through are small, the photons still loses energy.

Conclusion

We have shown for a system where a photon is scattered, via the process of Compton scattering, such that it traces out a regular polygon of n sides, polygons with less sides will change the wavelength of the photon much more significantly. This effect is less significant for successive polygons as difference in the exterior angles and hence the scattering angle for successive polygons decreases.

More research could be done to see if this effect can be observed experimentally, perhaps setting up the stationary electrons into their polygon configurations using Wigner crystals [2].

References

[1] Tipler, P. and Mosca, G. (2008). Physics for scientists and engineers. New York: Freeman.

[2] Wigner, E. (1934). On the Interaction of Electrons in Metals. Physical Review, 46(11), pp.1002-1011.