Journal of Physics Special Topics

An undergraduate physics journal

A4_6 Leap Years

J. Terry, N. Jinnah, A. Hogan, B. Henderson

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

November 10, 2018

Abstract

In this paper we assess the feasibility of using a rocket to push the Earth into an orbit with a period of exactly 365 days, thus eliminating the need for leap years. Assuming circular orbits, we find that moving the Earth into an orbit of the desired period would require a change in velocity of 6.3989 ms^{-1}. A rocket with a specific impulse of 400 seconds would need to consume 9.735×10^{21} kg of propellent to produce this velocity change, meaning that the idea is not feasible using current rocket technology.

Introduction

Because the Earth orbits the Sun with a period of 365.25 days, it is impossible to evenly divide the year into a whole number of days. This necessitates the existence of leap years, which correct for the remainder by adding an extra day, 29 February, to every fourth year. This solution is unsatisfactory, however, as it leads to an inconsistency in the calendar and causes issues whenever anniversaries such as birthdays fall on 29 February. Furthermore, it means that there is a period of time when a new calendar year has begun before the Earth has actually completed its orbit. There have been suggestions in the past for alternative calendars which aim to provide different solutions to the problem; however none of them have ever been widely adopted. Regardless, no calendar can eliminate the fundamental problem that the Earth's orbital period is not divisible by its period of rotation. Nevertheless, there may be a solution. By altering the orbital period of the Earth, it should be possible to reduce the length of one year to exactly 365 days, eliminating the need for leap years.

Method and Results

The relationship between the period, P and the semi-major axis, a of an orbit is given by Kepler's third law of planetary motion:

$$P^2 = \frac{4\pi^2 a^3}{G(M+m)}, \tag{1}$$

where M and m are the masses of the two orbiting bodies and G is the gravitational constant. If we assume that our initial and final orbits are both circular, then the semi-major axis is equal to the radius. Initially, the Earth orbits with a period of 365.25 days and a radius of 149.60×10^9 m [1]. Using equation 1, we determine that an orbital period of 365.00 days would correspond to a radius of 149.53×10^9 m.

The relationship between period and angular velocity, ω, is:

$$\omega = \frac{2\pi}{P}, \tag{2}$$

and angular velocity is related to linear velocity, v by:

$$v = \omega r, \tag{3}$$

where r is the radius. Hence, our initial period of 365.25 days corresponds to an initial tangential velocity of 29786 ms^{-1}, while our final period and radius correspond to a velocity of 29792 ms^{-1}. The difference between these two values, Δv, gives the change in the Earth's velocity needed to move from the initial to the final orbit. This was found to be 6.3989 ms^{-1}.

To determine whether it might be feasible to produce this change in velocity, we consider the impulse needed to generate the corresponding change in the Earth's momentum, as given by a variation of Newton's second law of motion:

$$I = m\Delta v, \tag{4}$$

where I is impulse and m is mass [2]. We calculate the impulse necessary to change the Earth's velocity by 6.3989 ms^{-1} as 3.8214 Ns.

We then utilise the concept of specific impulse, which is defined as:

$$I_{sp} = \frac{I}{m_p g_0}, \tag{5}$$

where m_p is the mass of propellant consumed and $g_0 = 9.81$ ms^{-2} is standard gravity [3]. Propellants in common use yield specific impulses in the range 175 to 300 seconds, while some propellants can theoretically produce a specific impulse of up to 400 seconds [4]. Using equation 5, we calculate values of m_p for each of these three values of I_{sp}; these are tabulated below.

I_{sp} (s)	m_p (kg)
175	2.23×10^{22}
300	1.30×10^{22}
400	9.74×10^{21}

Discussion and Conclusion

Our calculations indicate that the mass of propellant that would be needed for a rocket to change the Earth's orbit enough to eliminate leap years is prohibitively large; even the smallest of our calculated values of m_p is around 0.16% of the mass of the Earth. It is highly unlikely that enough suitable propellant could be gathered, and a very high mass flow rate would be needed to complete the burn in a reasonable period of time. This could be achieved either by using an extremely large rocket engine, or a large number of more conventionally sized engines working in concert Possible areas for further research could include attempting to determine whether other forms of propulsion might be more effective than rockets. There are types of propulsion which can produce greater specific impulses than chemical rockets, so it is possible that they might be able to complete the manoeuvre without the need for such a large mass of propellant. Future work in this area may also attempt to improve the accuracy of our results by eliminating the approximation of the orbits as circular; in reality the Earth's orbit is elliptical, however with an eccentricity of just 0.0167 [1] the deviation from a circular orbit is small.

References

[1] `https://solarsystem.nasa.gov/planets/earth/by-the-numbers/`[Accessed 29 October 2018]

[2] `http://zonalandeducation.com/mstm/physics/mechanics/momentum/introductoryProblems/momentumSummary2.html`[Accessed 29 October 2018]

[3] `https://www.grc.nasa.gov/www/k-12/rocket/specimp.html` [Accessed 28 October 2018]

[4] `https://history.nasa.gov/conghand/propelnt.htm` [Accessed 28 October 2018]