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P4_3 Magnetospheric Energy Harvest

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Abstract

It has been known since Michael Faraday's work on electromagnetism that passing a conductor through a magnetic field generates an electromotive force (EMF) across that conductor. This principle is applied to a conductor falling through the Earth's magnetosphere, which is modelled as a dipole field. In this report, the possibility of such a system powering the electronics of a satellite in a circular orbit is explored. A polar orbit (PO) is considered in the calculation of an EMF, which is induced across a theoretical 10 m long conducting rod. The EMF for the PO varies with latitude and so is only considered over the poles, where the B field direction is assumed to be constant. The plausibility of using this system as a power source for a satellite is discussed on the basis of the EMF values. The calculated EMF value for the PO is $\epsilon_p = 4.0$ V, which is time varying over a small range of EMF values. The altitude used for the PO is 400 km.

Introduction

If a satellite in a PO carries a conducting rod, the rod will cut through the Earth's magnetic field and generate an EMF, and therefore induce a current which may power on-board electronics. The magnetic field of the Earth is assumed to be that of a small dipole magnet, i.e. the poles are close together, which rests at the Earth's centre. The area swept out by the conducting rod, as the satellite runs it's orbit, is proportional to the magnetic flux that passes through it. The EMF is therefore proportional to the rate in which this area is swept out. The unit vector $\hat{n}$ is associated with this area and it is assumed to point parallel to the total B field. This is only true over small time intervals (with respect to the orbital period). The PO is only considered over such time scales at an altitude of 400 km which is the altitude of a low Earth orbit [1].

Theory

The magnitudes of the radial and azimuthal components for the magnetic flux density are given respectively below [2].

$$B_r = 2B_o cos\theta \left(\frac{R_e}{r}\right)^3 \tag{1}$$

$$B_\theta = B_o sin\theta \left(\frac{R_e}{r}\right)^3 \tag{2}$$

Where,

$$\theta = \omega t \tag{3}$$

B_o is the mean magnetic flux density on Earth at the magnetic equator, θ is the azimuthal displacement from magnetic north, R_e is the Earth's radius which is 6371 km [3], r is the radial displacement from the Earth's center, which is 6771 km. ω is the angular speed and is calculated from a consideration of the equations of circular motion and Newton's law of gravitation [4].

This calculation involves the gravitational constant G, and mass of the Earth which have values 6.67×10^{-11} m^{3}kg^{-1}s^{-2} [5] and 5.974×10^{24} kg [4] respectively. The total B field can be found by combining equations (1) and (2) using Pythagoras' theorem. The total B field is then,

$$B_p = B_o \left(\frac{R_e}{r} \right)^3 \sqrt{3(cos(\omega t))^2 + 1} \quad (4)$$

From Faraday's law, an expression for the EMF can be found. The version of Faraday's law which is useful in this case is below.

$$\epsilon = -\frac{d}{dt} \int_a^b \vec{B}.\hat{n} dA \quad (5)$$

Where, a is zero and b is related to the conductors' destination some time later. $\hat{n}$ is the direction vector of the area swept out by the moving conductor, and is parallel to $\vec{B}$ over short time intervals. The area element dA can be written as $lvdS$, where l is the length of the conductor (10 m), v is the tangential speed and dS is an element of arc length. v was calculated by considering the rod to move in a circular orbit and has the value of 7670 ms^{-1}. When the above is solved we get the following equation

$$\epsilon = \frac{3C\omega t cos(\omega t) sin(\omega t)}{\sqrt{3(cos(\omega t))^2 + 1}} - C\sqrt{3(cos(\omega t))^2 + 1} \quad (6)$$

Where C is a constant. The EMF was calculated over the poles and was plotted over a time interval of 100 s, which is small with respect to the orbital period. The orbital period can be calculated by dividing 2π by ω which gives a large 5546 s. Over such a small time interval, the change in field direction experienced by the conductor is small. The calculated EMF to 1 d.p. is 4.0 V. This value satisfies the voltage requirement of a radio transmitter [6].

Figure 1 shows how the EMF across the conductor varies with time. The conductor begins it's journey at magnetic north with speed v.

Conclusion

The values obtained for the EMF suggest that electronics which require a potential difference of 4 V, for example a radio transmitter [6], can in theory be powered by the system described in this report. This is only true for regions of the orbit in which a significant flux passes through the area swept out by the conducting rod. If $\hat{n}$ and $\vec{B}$ are perpendicular, which is true when the conductor passes over the equator, no EMF is generated. The PO, despite generating a large EMF at the poles, is not reliable for systems which require continuous power. An equatorial orbit will generate a constant EMF throughout the orbit due to the consistent orientation of the B field. This is an area for future study.

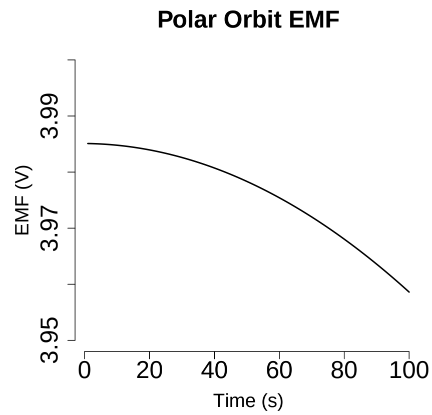

Figure 1: Graph of EMF as a function of time for a conductor in a polar orbit.

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