P3_2 The Power Of Rock

H. Conners, S. Lovett, P. Patel, C. Wilcox

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

This paper calculates an approximation for the power produced when a guitar string is plucked at different frequencies and how the amplitude varies the power generated. It was calculated that a frequency of 330 Hz and amplitude of 3 m would produce 5 W of power, which is enough to power a small amplifier.

Introduction

A pickup on an electric guitar consists of a group of usually 6 magnets (one for each string) and a coil of wire. A electromotive force (EMF) is produced by electromagnetic induction when the guitar is played due to the strings disturbing the magnetic field produced by the magnets. This EMF is then processed and heard through an amplifier. We shall calculate the current produced by a "Fender Vintage Noiseless Strat Pickups" [1] and the amplitude a string would need to be plucked to theoretically power an amplifier.

Theory

Faraday's law states that a rate of change of magnetic flux, $\frac{d\Phi_B}{dI}$ about a conductor will induce an internal EMF, V, which opposes the change. Faraday's law is shown in Equation 1.

$$V = -\frac{d\Phi_B}{dt} \tag{1}$$

For simplicity, we are assuming the guitar has a single pickup and a single string. Furthermore, the string completely disturbs the magnetic field (B=0) produced by the pickup as it passes directly above it and that the string vibrates as a standing wave. Change in magnetic flux can be expressed as in Equation 2.

$$d\Phi_B = A\Delta B \tag{2}$$

Where ΔB is the change in magnetic field and A the area through which the field is acting. The B field will change at a rate proportional to the frequency of the vibrating string. We shall consider 6 frequencies, which correspond to the E standard guitar tuning. The frequency, f is the number of completed cycles a second. The time period of one cycle is the inverse of frequency. In one cycle the string will travel a distance of 4 times the amplitude, a, whilst passing over the pickup and disturbing the field twice per cycle. If we assume acceleration is instantaneous we can calculate the velocity of the vibrating guitar string for a given frequency shown in Equation 3.

$$4af = v \tag{3}$$

The time taken over the pickup is seen to be dependent on frequency and amplitude of the string, and is expressed below.

$$dt_{pickup} = \frac{w}{4af} \tag{4}$$

Assuming the time taken for the string to disturb the field is equal to the time spent over the pickup, we can combine Equations 1, 2 and 4 to give the relationship shown in Equation 5.

$$V = -A\Delta B \frac{4af}{w} \qquad (5)$$

Using Ohms law and the definition of electrical power, we find that the power generated in the pick up is given by Equation 6.

$$P = IV = \frac{V^2}{R} = \frac{(A\Delta B 4af)^2}{w^2 R} \qquad (6)$$

Analysis

The constant magnetic field, B produced by the pickups is given as approximately 800 mT [2]. The resistance of the pickups is given as 9.8 kΩ [1]. Assuming the coil has the same dimensions as the rectangular pickup with a width, w and length, l of 18 mm and 70 mm respectively [3], giving an area, A of 1260 mm^2. Using these values along and those previously mentioned, along with Equation 6, a plot of Power against amplitude was produced, Figure 1 for all of the frequencies found in E standard tuning [4].
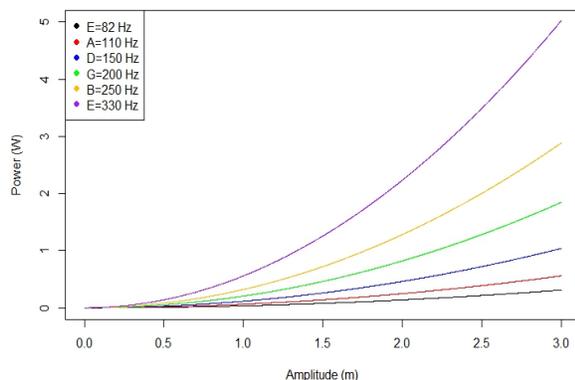

Figure 1: Power against amplitude plot for E standard tuning

As expected from Equation 6, a square law is found and higher frequencies generates more power.

Discussion

As seen in figure 1 the amount of power generated by the pickups is trivial, even at vibrations with amplitudes of 0.5 m, which is much greater than an amplitude required for a string to break. Ignoring this and using a frequency of high E (330 Hz) [4] and Equation 6, the amplitude required to power a 5 W amplifier would be about 3 m, which is considerably longer than any guitar or bass. It should be noted that in this calculation we have assumed that a guitar string passing over a pickup completely cancels the field it produces which may not be the case in reality. It could be considered that more than one string is often played at a time, however due to how little power is produced by this effect means it would still be impractical.

Conclusion

To conclude we found that it is an impractical idea to power a guitar amplifier by using the power generated in the guitars pickups whilst being played. An amplitude of 3 m would be required at a frequency of 330 Hz to power a small 5 W amplifier. This amplitude is longer than the guitar itself and would cause the strings to break. For further research, more strings could be considered to be played simultaneously, as well as more sensitive pickups could be investigated.

References

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