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A6_3 Taking the Moon to Mars

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Abstract

We apply the 'Method of Patched Conics' to estimate the trajectory the Moon would take if it were to be removed from its current orbit and placed in the same orbit (as it is placed at Earth) at Mars. We calculate that the velocity changes at Earth and Mars required for such a trajectory are 1906 ms^{-1} and 1356 ms^{-1} respectively. We determine that it would take one coal burning power plant in excess of 1×10^{12} years to produce enough energy for just one velocity change, making the concept predictably unfeasible with current technology.

Introduction

We will apply the 'Method of Patched Conics' [1] to the Moon in an attempt to calculate the velocity (hence the kinetic energy) changes required to depart its geocentric orbit and be placed in a martian orbit. The Method of Patched Conics separates problems into phases that can be analysed independently before 'patching' results together. We will use three phases in our calculations: a i) Geocentric phase, ii) Heliocentric phase, and iii) a 'Mars-centric' phase. Phases i) and iii) are bound by the Sphere of Influence (SOI) of the Earth and Mars respectively - that is the area in which they are the dominant bodies of the Moon's motion. An illustration of the proposed trajectory can be found in Fig. 1. We will make several key assumptions to simplify the problem further: 1) The SOI sizes are negligible compared to the orbital radii of the planets around the sun, 2) The Moon's mass is negligible compared to planetary masses, 3) the planets lie on the same plane and will be suitably aligned (as shown in Fig. 1), 4) the Moon begins and ends with the same circular orbit and 5) all velocity changes are instantaneous.

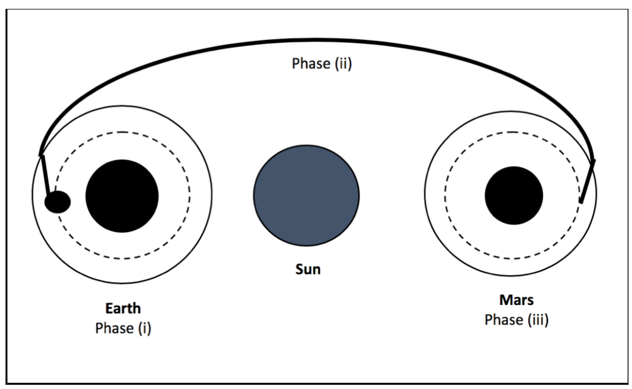

Figure 1: Schematic illustrating the proposed trajectory. The trajectory is represented by the bold line, the Spheres of Influence are shown by the solid lines around each planet, and the initial and final orbits of the Moon are highlighted by the dashed lines. The phases used in our calculations are also labelled.

Method

We used the Vis-Viva equation (1) at various stages of the Moon's trajectory to determine the velocity changes required to transfer it into a martian orbit.

$$v = \sqrt{\frac{2GM}{r} - \frac{GM}{a}} \tag{1}$$

The Vis-Viva equation gives the velocity, v, in terms of its radial position (distance from the central body), r, the semi-major axis of the orbit, a, and the mass of the dominant body M. The gravitational constant is represented by G.

We first calculated the velocity change, Δv, required at Earth to place the Moon in a heliocentric (elliptical) orbit (HCO) to Mars. We applied a hyperbolic trajectory to transfer the Moon from its initial circular orbit to the HCO. Hyperbolic trajectories are unbound and provide fast transfer between orbits. The heliocentric velocity at Earth's SOI, $v_{HC,E}$, was calculated using (1) for $M = M_{Sun}$,

$$a = \frac{R_E + R_M}{2}, \tag{2}$$

where R_E and R_M are the orbital radii of the Earth and Mars respectively (1 au and 1.5 au) and $r = R_E$. To place this velocity in the reference frame of the Earth, we subtracted the orbital velocity of the Earth ($\approx$ 30,000 ms^{-1}) from the calculated value. As the SOI and initial orbit must lie on the same hyperbolic transfer orbit, we applied a rearranged form of (1) to determine the required value of semi-major axis (for the transfer orbit). We used values of $M = M_{Earth}$, $v = v_{HC,E}$ and $r = R_{SOI,E}$ - the radius of the Earth's SOI ($\approx$ 0.1 au). With the appropriate value of a, we then calculated the velocity the Moon would have on the hyperbolic trajectory at the radius of its initial orbit, $v_{H,C}$. The required velocity change was then simply

$$\Delta v = v_{H,C} - v_C, \tag{3}$$

where v_C is the initial circular velocity of the Moon's orbit ($\approx$ 1000ms^{-1}). After changing the appropriate constants (mass etc.), we used an identical method to calculate the velocity change required to place the Moon in its new circular orbit around Mars. The energy required for each velocity change was calculated using

$$\Delta KE = \frac{1}{2} M_m \Delta v^2, \tag{4}$$

where M_m is the mass of the moon ($\approx 7.35\times10^{22}$ kg).

Results and Discussion

We found that the velocity change required to remove the Moon from its geocentric orbit was $\Delta v_E = 1906$ ms^{-1} and to place it in the same orbit around Mars was $\Delta v_M = 1356$ ms^{-1}. The energies required to produce these velocity changes were 1.34×10^{29} J and 6.76×10^{28} J respectively. Even if it were possible to create a propulsion device to send the Moon to Mars, these energy requirements make the process unfeasible. For instance, given that the Drax coal burning power plant in the UK produces 4000 MW of power [2], we calculated it would take over 1×10^{12} years to produce enough energy just to remove the Moon from its current geocentric orbit using such a source.

Conclusion

Unfortunately, perhaps predictably, it is very unlikely we will be able to take the Moon to Mars. Our results have quantified the nature of the problem, and shown the currently unfeasible energy costs propulsion devices must overcome in the future if this concept were to become reality.

References

[1] H. Curtis, *Orbital Mechanics for Engineering Students* (Elsevier Aerospace Engineering Series, 2005), p. 359.

[2] `https://www.mpoweruk.com/fossil_fuels.htm` [Accessed 6 October 2018]