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P4_2 Parachuting in Tibet

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Abstract

It is well known that the density of air decreases with altitude. This article discusses the minimum cross-sectional area of a parachute required for a safe landing on the Tibetan Plateau (4500 m in altitude). We calculated that a cross-sectional area of 34 m^2 is needed to limit the deceleration down to a safe 16 G, whereas, at sea level the cross-sectional area needed is only 22 m^2.

Introduction

A parachute works by creating a large cross-sectional area (henceforth refered to a CSA), which increases the drag and therefore reduces the terminal velocity of the parachutist. Drag is proportional to the CSA, so changing the CSA of the parachute will affect the terminal velocity and hence the deceleration of the parachutist. If the deceleration is greater than 16 G (16 times the acceleration due to gravity on the Earth's surface [9.81 m s^{-2}]) there is a chance that the parachutist will be injured [1]. In this report we calculate the minimum CSA of a parachute required to achieve a deceleration of 16 G at 4500 m, which is the average height of the Tibetan Plateau [2].

Theory

The parachutist reaches terminal velocity when the drag on the parachute is equal to the weight of the parachute and parachutist combined. The drag due to the parachute is given by [3]:

$$D = \frac{1}{2}C\rho A v^2 \tag{1}$$

Where C is the drag coefficient of the parachute, ρ is the density of air, A is the CSA of the parachute and v is the velocity at which the parachutist is falling. The weight is given by:

$$W = (m_p + m_m)g \tag{2}$$

Where m_p and m_m represent the mass of the parachute and the mass of the parachutist respectively and g is the gravitational field strength (9.81 m s^{-2}). Equating equations (1) and (2), and rearranging for A gives:

$$A = \frac{2(m_p + m_m)g}{Cv^2\rho} \tag{3}$$

To calculate the density of the air at different altitudes, we assumed that air is an ideal gas [4] in hydrostatic equilibrium [4]. Therefore,

$$\frac{\partial p}{\partial h} = -\rho g \tag{4}$$

$$\rho = \frac{p}{RT} \tag{5}$$

Where p is the pressure, h is the altitude, R is the specific gas constant of air which is 287 J kg^{-1} K^{-1} [5] and T is the temperature of the air. The rate of temperature decrease in the troposphere L is 0.0065 K m^{-1} [6], the troposphere extends to

10 km above the ground; as the Tibetan Plateau is only 4500 m high we do not need to concern ourselves with anything beyond the troposphere. Therefore, the temperature of the air and the change in altitude are given by:

$$T = T_0 - Lh \tag{6}$$

$$\partial h = -\frac{\partial T}{L} \tag{7}$$

Combining equations (4), (5) and (7) gives:

$$\frac{\partial p}{p} = \frac{g}{RTL}\partial T \tag{8}$$

Integrating the left and right hand sides of equation (8) between p_0 and p, and T_0 and T respectively, solving for p and substituting into equation (5) gives ρ as a function of altitude h. Where p_0 is the atmospheric pressure at sea level (101 kPa or 1.01×10^5 Pa) [4] and T_0 is the temperature at sea level (288 K) [4]. Finally, replacing T with equation (6) gives:

$$\rho = \frac{p_0}{R(T_0 - Lh)}\left(1 - \frac{Lh}{T_0}\right)^{\frac{g}{LR}} \tag{9}$$

Results

Substituting the appropriate values into equation (9) allows us to find the density of the air at any altitude h. Therefore, we calculated that the density of air on the Tibetan Plateau (h of 4500 m) is 0.77 kg m^{-3}, compared to 1.2 kg m^{-3} at sea level (h of 0 m).

To calculate the maximum safe terminal velocity we needed to find the maximum deceleration that a human could withstand. According to Kumar and Norfleet [1] the human acceleration tolerance limit for a feet first impact is 16 G for up to 0.04 s. These values give maximum terminal velocity of 6.3 m s^{-1}.

Substituting this value of the terminal velocity into equation (3) as well as a value for ρ from equation (9), we could calculate the minimum CSA of the parachute needed to land safely at different altitudes. For this we assumed that the total mass of the parachute and parachutist is 80 kg and that the parachute is dome-shaped so the drag coefficient of the parachute C is 1.5 [7].

We found that the CSA needed to land safely on the Tibetan Plateau is 34 m^2. Whereas, we calculated the CSA needed to land safely at sea level to be 22 m^2. If you were to use the same CSA as is needed at sea level when landing on the Tibetan Plateau the terminal velocity would be 7.9 m s^{-1}, this would mean a deceleration of 20 G, which is likely to injure the parachutist [1].

Conclusion

The change in density with altitude makes a huge difference to the minimum size of a parachute needed to safely land at different altitudes. To land safely on the Tibetan Plateau which has an elevation of 4500 m, a parachutist would require a parachute with a CSA of 34 m^2 which corresponds to a radius of around 3.3 m. Whereas, to land safely at sea level a parachutist would require a parachute with a CSA of 22 m^2.

References

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