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P6_2 Music Under the Sea

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Abstract
We consider the differences in how music would sound to Ariel from Walt Disney's The Little Mermaid when on land compared to under the sea. We take into account the dependence on temperature and depth when calculating the speed of sound in salt water. We found that Ariel's voice would be higher in frequency when under the sea, along with wind and brass instruments. In order to sound familiar to her on land, these instruments would have to be shortened in length. Percussion and string instruments would not be affected.

Theory

Sound travels at different speeds through different mediums. Sound waves are longitudinal waves which consist of compressions and expansions. The wave speed is dependent on how fast it bounces back from each compression. The less compressible the medium, the faster it bounces back. Water is less compressible than air; therefore sound has a higher speed in water than air. Values for the speed of sound are found using:

$$\nu^2 = \frac{B}{\rho} \tag{1}$$

Where ν is the speed of sound, B is the bulk modulus (a measure of how compressible a medium is) and ρ is the density of the medium. We found values for the speed of sound in air and pure water, as shown in table 1. Taking the note middle C which has a frequency of 261.6 Hz [3], a value for the wavelength of this note in air is found to be 1.11 m using:

$$\nu = f\lambda \tag{2}$$

Where ν is the speed of sound, f is the frequency and λ is the wavelength.

	Bulk modulus [1] $\mathrm{kNm^{-2}}$	Density [2] $\mathrm{kgm^{-3}}$	Speed $\mathrm{ms^{-1}}$
Air	101	1.20	290
Water	2.12×10^6	1000	1470

Table 1: Speed of sound in air and pure water

Ariel lives in the Ocean however, so the water is salt water. This means that the temperature, t, depth, d, and salinity, s, also have to be considered. There are many different equations for the speed of sound in salt water; the one used here is Mackenzie's [4]. We took the salinity to be a constant value of 35 parts per thousand (PPT), which is the average salinity of Earth's oceans [5]. Using this value also means that two terms in Mackenzie's equation are zero and can therefore be ignored giving:

$$\begin{aligned} c(d,s,t) = \; & 1448.96 + 4.591t - 5.304 \times 10^{-2}t^2 \\ & + 2.374 \times 10^{-4}t^3 + 1.630 \times 10^{-2}d \\ & + 1.675 \times 10^{-7}d^2 - 7.139 \times 10^{-13}td^3 \end{aligned} \tag{3}$$

Where temperature is in degrees Celsius and depth in meters. Eq. (3) is valid in the range: $2\,°\mathrm{C} < t < 30\,°\mathrm{C}$ and $0\,\mathrm{m} < d < 8000\,\mathrm{m}$. Because the temperature and depth vary in the ocean, three values were taken for depth with their corresponding value for temperature as shown in table 2. The values were found from figure 1 using the red line which is of latitude between -15 ° South to 15 ° North. Using Eq. (3) along with

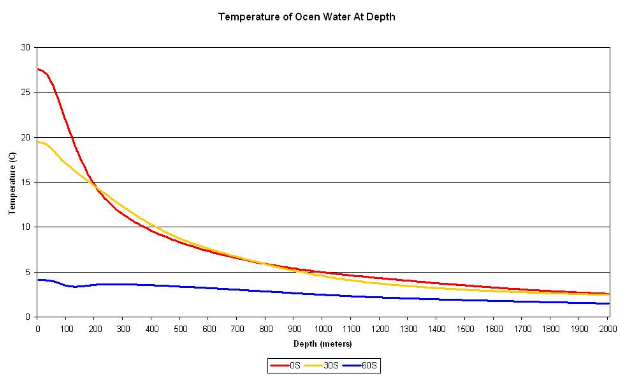

Figure 1: Temperature of ocean water depth [6]

Depth m	Temperature °C	Speed ms^{-1}	Frequency Hz
0	27.5	1540	1390
1000	5.0	1487	1340
2000	2.5	1493	1350

Table 2: Speed of sound in salt water

the values for depth and temperature, the values for speed of sound at the various depths were found as shown in table 2. Using these speeds and the wavelength of 1.11 m (the value needed to play the note middle C in air) in Eq. (2), the frequencies shown in table 2 were found.

If Ariel wanted wind and brass instruments to sound familiar above the sea, a change would have to be made to the instrument. The wavelength is determined by the length of these instruments, so it's fixed. Modelling them as a simple pipe closed at one end [7],

$$f = \frac{\nu}{\lambda} = \frac{n\nu}{4l} \quad , \quad \left(l = \frac{n\lambda}{4} \right) \tag{4}$$

,where l is the length and $n = 1, 3, 5, \ldots.$

This would mean that the instruments would have to be shortened to produce the same frequency in air as a regular instrument would in water.

Conclusions

It was found that the frequencies produced under the sea were over five times higher than those in air, although the exact difference varies depending on depth. This means that Ariel would be used to a higher pitch voice than she would experience once she is on land. Wind and brass instruments would also be affected in the same way with the frequency being higher under the sea. Hence, if Ariel wanted the music to sound familiar she would have to shorten the instruments as shown by applying Eq. (4). Other instruments, such as the harp or drums, would not be affected as the sound waves are produced from vibrations.

References

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