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A1_6 The Ultimate Curveball

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Abstract

Drag effects play an important effect in everyday life, but is most noticeable in ball games such as baseball and football. In this paper, we determine the initial velocity and rotational velocities we need to set a ball at in order for the effects of spin to allow for it to come back around to us. This is done by considering the Magnus effect. We find that these properties are proportional to each other, but the relationship between the object's mass and radius plays a bigger role than expected.

Introduction

The Magnus force is a drag force, resulting due to the object's spin. Air moves faster on one side than the other, allowing for seemingly gravity defying tricks. By considering the different drag forces on the particle and obtaining equations of motion, we can find the required initial conditions to throw a ball and have it return to you as a result of its spin.

Theory

The Magnus force acts in the direction perpendicular to the velocity and the axis of rotation of the ball, with magnitude [1]:

$$F_M = \frac{4}{3}(4\pi^2 {r_b}^3 \omega_b \rho_{air} v) \tag{1}$$

Where r_b is the radius of the ball, ω_b is the rotational velocity of the ball, or its spin and v is the velocity of the ball. We consider the scenario in which we pitch the ball parallel to the horizontal (xy plane). We assume that we can treat components in z and xy independently, since the Magnus force in the z-direction is negligible compared to the force of gravity, i.e. $F_{M,z} = 0$.

First, we consider the deceleration of the ball as it travels around in the xy plane, which is just the drag force. From Newton's Second Law,

$$\frac{1}{2}\rho_{air} v^2 C_D A = ma \tag{2}$$

Where C_D is the drag coefficient and A is the cross-sectional area. Re-arranging this gives us a differential equation in v:

$$\beta v^2 = \frac{dv}{dt} \implies \beta = \frac{\rho_{air} C_D \pi {r_b}^2}{2m} \tag{3}$$

Solving Eq. (3) and substituting for our initial conditions, i.e. at $t = 0$, $v = v_0$:

$$v(t) = -\frac{1}{\beta t - \frac{1}{v_0}} \tag{4}$$

We assume that the point about which the motion is circular is constant. Additionally, we assume ω_b is constant with time, i.e. it does not decay. The Magnus force acts as a centripetal force, so using Eqs. (1) and (4):

$$\frac{4}{3m}[4\pi^2 {r_b}^3 \omega_b \rho_{air} v(t)] = \frac{v(t)^2}{r(t)} \tag{5}$$

$$\gamma\omega_b = \frac{1}{r(t)}\frac{1}{\beta t - \frac{1}{v_0}} \implies \gamma = \frac{16\pi^2 {r_b}^3 \rho_{air}}{3m} \quad (6)$$

We assumed $F_{M,z} = 0$ and $v_{0,z} = 0$, so we have $t = \sqrt{\frac{2h}{g}}$, where h is the height over which the ball falls. Substituting in Eq. (6):

$$\omega_b = \frac{1}{\gamma r(t)}\left[\frac{1}{\beta\sqrt{\frac{2h}{g}} - \frac{1}{v_0}}\right] \quad (7)$$

If we specify t, then $r(t)$ gives the radius from centre of rotation of the ball's motion, which is an outwards spiral.

If we specify a radius around us that we can catch the ball from, then we simply equate $r(t)$ to the equation of this circle. By using the equation of an off centre circle in polar coordinates, this expression is:

$$r(t) = \sqrt{{r_c}^2 - r(0)^2 - 2r(t)r(0)cos(\theta - \phi)} \quad (8)$$

Where r_c is the maximum radius at which we can catch the ball and ϕ is the angle from the origin to some point on the circle centred around us. For simplicity, take $\phi = 0$. Also, combining Eqs. (1), (4) and (5) taking $t = 0$ gives $r(0) = \frac{v_0}{\gamma\omega_b}$. Taking these expressions into Eq. (7):

$$\omega_b = \frac{v_0}{r_c}\left[\frac{1}{\gamma\left(\beta v_0\sqrt{\frac{2h}{g}} - 1\right)^2} + 1\right] \quad (9)$$

We have 6 parameters: 3 to do with the ball (r_b, C_D, m) and 3 that are not (ρ_{air}, h, r_c).

Results

The parameters that are not associated with the ball can be held constant: $\rho_{air} = 1.225$ kg m^{-3} [2], $h = 1$ m (so that we can catch the ball) and $r_c = 0.8$ m, the average arm length.

We can try different values for the parameters of the ball, however. Taking values for a baseball, a tennis ball [4] and a football [5], assuming that velocities are high enough to be in turbulent flow, i.e. $C_D = 0.47$ [6], we produce the following plot.

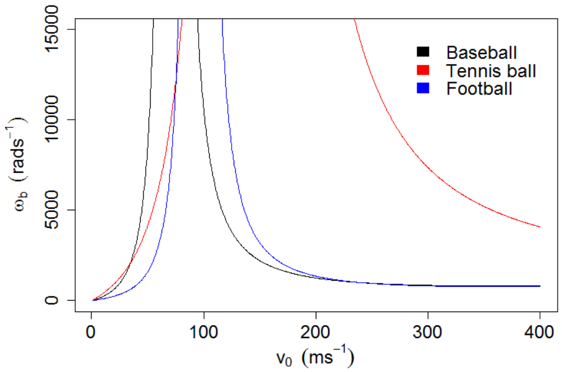

Figure 1: Plots of ω_b against v_b for a baseball (r_b = 0.036 m, m = 0.035 kg) a tennis ball (r_b = 0.033 m, m = 0.056 kg) and a football (r_b = 0.11 m, m = 0.43 kg).

Discussion and Conclusion

As we expect, as we increase v_0, ω_b must also increase. However, this does not take into account the range of ϕ we can have, since we can catch the ball everywhere inside our circle of radius r_c.

We can see that the values all have some peak. This is because there are few v_0 and ω_b that produce a spiral that corresponds to the position of the initial throw. Other combinations simply reach this point more slowly , relatively speaking.

Additionally, there is a balance between the mass and the radius of the object. The baseball and tennis balls have similar mass, but the change in radius makes a noticeable difference. This relationship could be explored in a future paper.

References

[1] https://goo.gl/zu4En2 [Accessed 29. Nov 2017]

[2] https://goo.gl/Gu33Cw [Accessed 29. Nov 2017]

[3] https://goo.gl/7NYbjD [Accessed 29. Nov 2017]

[4] https://goo.gl/8UbKvn [Accessed 29. Nov 2017]

[5] https://goo.gl/LYi2Fc [Accessed 29. Nov 2017]

[6] https://goo.gl/ZVw942 [Accessed 29. Nov 2017]