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S1_6 The Wings of Christmas

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Abstract

According to traditional lore, each Christmas Santa Claus travels across the globe delivering gifts to all of the children on his 'Nice' list. In this paper, we calculated the velocity he would travel at to be 1.56×10^6 ms^{-1}. Thus, if his sleigh had aerofoils they would require a minimum surface area of 1.26×10^{-3} m^2 each to provide lift.

Introduction

The traditional model of Santa Claus is inspired by Saint Nicholas, who comes from the Christian religion [1]. It is said he flies around the globe, on a sleigh driven by nine reindeer on Christmas eve night. He delivers gifts to the Christian children of the world while they sleep. We calculated the velocity at which he must travel to do this in one night and the size of the aerofoils required.

Theory

In order to find the velocity at which Santa travels, we first needed to find the distance he must fly. As Santa delivers to only the Christian children, we calculated the number of households he must visit during the night. This was done by multiplying the total population of children by the percentage of Christians in the world. The average fertility rate, or child per household, is 2.5 [2], which may be rounded to 3 for the purposes of our calculation. Dividing the total number of Christian children by 3 gives the number of households these children occupy, N_h.

The distance between these houses was found by dividing the Earth's surface area by N_h to give the area occupied by each household and half the distance to its nearest neighbours, A_h. If they are distributed as square areas and assumed to be adjacent to one another with the house in the centre of the square, the shortest distance between each house, L, is given by taking the square root of A_h.

The surface area of the globe, A_s, was found using $A_s = 4\pi R^2$. Where R is the radius of Earth. Given roughly only 30% of Earth's surface is covered by land mass [3], A_h may be expressed as

$$A_h = \frac{0.3A_s}{N_h} = \frac{6\pi R^2}{5N_h}. \tag{1}$$

The total distance Santa must travel to deliver his gifts, D, is given by the sum of lengths between households;

$$D = L(N_h - 1). \tag{2}$$

The time for him to make his deliveries is 10 hours, assuming children are asleep between the hours of 8pm and 6am on Christmas eve. Taking time zones into account and presuming Santa travels from east to west, he has an additional 24 hours for the task. This gives him a time $T = 34$

hours = 122400 s. His velocity, v, can then be found by

$$v = \frac{D}{T} = \sqrt{\frac{6\pi R^2}{5N_h}}\frac{(N_h - 1)}{T}. \quad (3)$$

To find the minimum surface area of the aerofoils needed, the weight of Santa, his sleigh, the gifts and his reindeer were equated to the lift equation [4];

$$Mg = \frac{\rho v^2 A C_l}{2}. \quad (4)$$

Where M is the total mass, g is the acceleration due to gravity, ρ is airflow density, A is the combined aerofoil area and C_l is the coefficient of lift. Substituting $C_l = 2\pi\alpha$ [4], where α is angle of attack; the angle of the direction of the aerofoil in relation to the airflow, and rearranging for A, gave

$$A = \frac{Mg}{\rho v^2 \pi\alpha}. \quad (5)$$

Results

There are 2.2 billion children on the planet [5], and with 32.5% of world recorded as Christian [6], Santa must provide presents to 7.15×10^8 of them. This gave $N_h = 2.38 \times 10^8$ households. Using $R = 6371$ km [7], v was found to be 1.56×10^6 ms^{-1}.

M was found by summing the masses of all bodies involved. Santa, with his thick clothing to keep him warm, was given an estimated mass of 120 kg. The sleigh, assumed to be strong enough via magic to support the necessary weight, was estimated as 200 kg. To provide the forward velocity Santa has the help of 9 magical reindeer. The mass of an adult male reindeer can be up to 210 kg [8], which gave a combined reindeer mass as 1890 kg. Finally, by using an average weight of 1 kg per gift the combined weight of the gifts was 7.15×10^8 kg. Hence, $M = 7.15 \times 10^8$ kg.

The density of air used was that of sea level and at 15°C; $\rho = 1.225$ kgm^{-3} [9] and $\alpha = \frac{17\pi}{180}$ radians was used to provide maximum lift [10]. Using $g = 9.81$ ms^{-2}, Equation 5 gave the minimum surface area to support Santa's sleigh with his gifts and reindeer as 2.53×10^{-3} m^2. Therefore, each aerofoil only requires a surface area of 1.26×10^{-3} m^2.

Conclusion

We calculated the velocity Santa would need to travel to visit all of the Christian children of the world in time for Christmas morning. This was found to be 1.56×10^6 ms^{-1}. From this we found the minimum surface area each aerofoil would need to support the weight of the Sleigh, presents, reindeer and Santa. This was calculated as 1.26×10^{-3} m^2 each. There are some limitations to the calculations in this paper, for instance, we have only considered a scenario where the Sleigh's velocity is constant. A more in depth study could look at the acceleration of the sleigh and also the time spent at each household. Moreover, we would expect Santa to commute to and from the North pole and not start from the first house, therefore, this scenario could be investigated.

References

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