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A1_5 Hiding in Plain Sound

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Abstract

This paper aims to determine if diverting light with sound waves is practical. This is done by constructing a model for a speaker power output based on mitigating drag. We then use the results of "A1_4 Bending light with sound"(Warford et al., 2017) to determine whether the power required is plausible. We find that the power output required is $1.501(\frac{a}{\sqrt{2}})^3$ MWm^{-2}, where a is the radius in terms of the displacement amplitude; this is within human possibility, but not within practical application ranges.

Introduction

The paper "A1_4 Bending light with sound" (Warford et al., 2017) gives a sound wave requirement for the bending of light. Utilising these requirements, we treat the wave like a series of optical fibres and work out if any kind of light-path manipulation is plausible.

Theory

If we treat each space between two peaks in a sound wave as optical fibres, then in order to change the path of light, the sound wave-fronts need to provide a curved path around which the light can travel. According to the Huygens-Fresnel principle, this can be done with an aperture size equal to, or smaller than, the wavelength.

However, due to divergence, the intensity of the wave (I), and hence the pressure amplitude (P_0), will decrease with distance from the source (r); this will result in a decreasing refractive index. This can be reduced using funnels, however the final opening must be larger than the wavelength of the sound. The easiest minimisation to achieve would be reducing the divergence by a factor of a half (i.e. direct all the sound in front of the speaker).

The intensity of a sound wave is proportional to the square of its pressure amplitude. The intensity diverges as such:

$$I(r) \propto \frac{1}{\Omega r^2} \propto P_0(r)^2 \implies P_0(r) \propto \frac{1}{r}\sqrt{\frac{1}{\Omega}} \quad (1)$$

Where Ω is the solid angle the sound is projected into from the source.

In order to relate this to the properties the speaker outputs, we will consider the P_0 at the aperture:

$$P_0(r_s) \propto \sqrt{\frac{1}{\pi r_s^2}} \therefore \frac{P_0(r)}{P_0(r_s)} = \frac{r_s}{r}\sqrt{\frac{\pi}{\Omega}} \quad (2)$$

Where r_s is the radius of the aperture (assuming circular).

In order to find how physically plausible the case is, we will consider the power required for the speaker to produce a wave. We will model the speaker as a thin plate oscillating in air.

We assume that the energy associated with moving the plate itself is negligible in comparison to the drag force acting on the plate (plate's mass is small). The power is now what is required to maintain the waveform in the presence of drag. Thus, the equations of motion are:

$$x(t) = \Delta x sin(\omega t) \therefore x'(t) = \omega \Delta x cos(\omega t) \quad (3)$$

$$\implies x'(t) = \Delta x \omega \cos\left[arcsin\left(\frac{x(t)}{\Delta x}\right)\right] \quad (4)$$

With the drag force,

$$F_d = \frac{C_d \rho A v^2}{2} = \frac{C_d \rho A}{2} x'(t)^2 \quad (5)$$

Where C_d is the drag coefficient and A is the cross sectional area. Also, the density is treated as constant in this case, to give a maximum estimate.

We can get the average power from these equations by integrating the force over the distance to get the work done, and then divide that by the relevant time interval.

Bearing in mind the symmetries of the sinusoidal waveform, we need only consider the case between $0 \leq t \leq \frac{1}{4f}$, $(0 \leq x \leq \Delta x)$. Thus, using Eq.(4) and Eq.(5):

$$\Delta E = \int_0^{\Delta x} F_d \, dx = \frac{C_d \rho A}{3} \Delta x^3 \omega^2 \quad (6)$$

And so,

$$p_{avg} = \frac{\Delta E}{\Delta t} = \frac{16\pi^2 C_d \rho A}{3} \Delta x^3 f^3 \quad (7)$$

Where p_{avg} is the average power.

Results and discussion

To get the power required for the standard case discussed in Warford et al., we use Eq.(7) and substitute in the values from that case, $\Delta x = 1.57$ m and $\Delta x = 6.37$ Hz, to get the power. We will consider the power required per unit area of speaker.

Taking a drag coefficient of a plate at $C_d = 1.98$ and the density of air as 1.225 kgm^{-3}, we get $p_{avg} = 1.501$ MW m^{-2}.

When considering divergence, we can use Eq.(2) to find the power required to get the same effect a certain distance away. If we take $\Omega = 2\pi$; r_s as half one wavelength (Huygens' principle), $r_s = \frac{4\Delta x}{2}$; and consider $r = a\Delta x$:

$$P_0(r) = P_0(r_s)\frac{r_s}{r}\sqrt{\frac{\pi}{\Omega}} = P_0(r_s)\frac{\sqrt{2}}{a} \quad (8)$$

Since, (from Warford et al. [1])

$$P_0(r_s) = 2\pi f \rho v \Delta x, \therefore p_{avg} \propto P_0(r_s)^3 \quad (9)$$

Then we can say, to maintain the same n_0 at a distance $a\Delta x$ away, the power needs to increase by:

$$\frac{p_{avg}(r)}{p_{avg}} = \frac{a^3}{2\sqrt{2}} \quad (10)$$

So, we see that we need to increase $f\Delta x$ by a factor of $\frac{a}{\sqrt{2}}$. Thus, the power will increase by a factor of $(\frac{a}{\sqrt{2}})^3$. So, for $a = 3$, $p = 9.55 \times 1,501,000 = 14.3 \times 10^6 = 14.3$ MWm^{-2}.

In either case the power requirements are huge per unit area. They may become more plausible with a very small area speaker, however, it will still require a lot of power to be possible.

Conclusion

In conclusion, this paper has found that the power required for a speaker that bends light would be relatively high. However, here we have treated density as constant, which will not be the case; this approximation gets worse the faster the speaker has to travel, as air doesn't instantly enter the space the speaker has cleared. This will decrease our result for power a substantial amount.

Further research could be done into the acceptance angles of these cases, a more thorough analysis of the power requirements and the loss at bends.

References

[1] Warford, L., Hophuong, O., Li, L., Wu, T. (2017). A1_4 Bending light with sound, *Journal of Physics Special Topics*, Leicester