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P6_6 Vacuum Resistance Effects on Photon Sails

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Abstract

The total change in momentum from collisions between a spacecraft and interstellar hydrogen atoms was considered in order to calculate the the time taken for the spacecraft's velocity to half ($t_{\frac{1}{2}}$), and the total distance the spacecraft travels in this time ($s_{\frac{1}{2}}$). It was found that for Voyager 1, $t_{\frac{1}{2}} = 1.521 \times 10^{12}$ years and $s_{\frac{1}{2}} = 2.986 \times 10^7$ ly. For a Graphene Photon Sail, $t_{\frac{1}{2}} = 8.7$ years and $s_{\frac{1}{2}} = 0.54$ ly. It is concluded that the results are significant enough to merit further investigation.

Introduction

With the potential of Solar Sail missions to other stars in the near future, it is important to reconsider whether physicists can continue to neglect the effects of resistance caused by particles in space. In this paper, we will calculate the resistance that particles located in interstellar space have on two example spacecraft, Voyager 1 and a theoretical Photon Sail and how this effect's their velocities with respect to time and distance.

Theory

We consider a spacecraft, of forward facing surface area A , velocity v and mass M_s . The particles that populate space are mostly Hydrogen atoms of mass m_h [1], a craft will collide with N atoms per second along its path. By this definition, we can define $N = A\rho v$. Where v is the velocity of the spacecraft and ρ is the number density of the particles. The total change in momentum of the collisions can be expressed as:

$$\Delta M_s v = N \Delta m_h v_h^2 \quad (1)$$

Where v_h is the velocity of a hydrogen atom

after the collision. We assume there is no mass change over time. When the spacecraft collides with a particle, it is assumed a maximum momentum transfer occurs, meaning v_h increases from 0 to v . We can then state:

$$\frac{dv}{dt} = A\rho \frac{m_h}{M_s} v^2 \quad (2)$$

We can evaluate the implicit integration of equation 1 for a spacecraft, with respect to velocity and time between it's the initial and final velocities. As a result, we can acquire a function for t with respect to the ship's Δv .

$$t = \frac{M_s}{A} \frac{1}{\rho m_h} \left(\frac{1}{v_f} - \frac{1}{v_0} \right) \quad (3)$$

$t \rightarrow \text{inf}$ when $v_f \rightarrow 0$. To find a more useful result, we can calculate $t_{\frac{1}{2}}$, the time taken for the spacecraft to reach $v_f = \frac{1}{2}v_0$. This is calculated by the following:

$$t_{\frac{1}{2}} = \frac{M_s}{A} \frac{1}{\rho m_h} \left(\frac{2}{v_0} \right) \quad (4)$$

If we want to calculate $s_{\frac{1}{2}}$, the total distance travelled by a space craft in time $t_{\frac{1}{2}}$, we start

with the following definition:

$$\frac{dv}{dt} = \frac{dv}{ds} \frac{ds}{dt} = \frac{dv}{ds} v \quad (5)$$

By substituting for $\frac{dv}{dt}$ (equation 2), and by using a second implicit integration with respect for velocity and displacement, we can show that:

$$s = \frac{M_s}{A} \frac{1}{\rho m_h} \ln\left(\frac{v_0}{v_f}\right) \quad (6)$$

We once again substitute $v_f = \frac{1}{2}v_0$ to obtain:

$$s_{\frac{1}{2}} = \frac{M_s}{A} \frac{1}{\rho m_h} \ln(2) \quad (7)$$

Therefore we are able to describe the motion of a spacecraft using equations 4 and 7.

Results

The first spacecraft to be analysed is Voyager 1, which recently entered interstellar space. For the craft, $M_s = 733 \text{ kg}$ estimated $v_0 = 17000 \text{ m s}^{-1}$ [2]. Voyager 1 has a high-gain antenna dish of radius 3.7 m [3], therefore A is calculated as 10.75 m^2 , assuming it is a flat circle.

Space is not empty, it is filled with a low amount of hydrogen atoms of number density $\rho = 1 \times 10^5 \text{ m}^{-3}$ [1]. The mass of a Hydrogen atom is $m_h = 1.673 \times 10^{-27} \text{ kg}$ [4].

We applied equation 4 and 7 and found that for Voyager 1:

$$t_{\frac{1}{2}} = 4.795 \times 10^{19} \text{ s} = 1.521 \times 10^{12} \text{ years}$$

$$s_{\frac{1}{2}} = 2.825 \times 10^{23} \text{ m} = 2.986 \times 10^7 \text{ ly}$$

The second craft to be analysed is a Graphene based Photon Sail design. It would function similarly to Solar Sail, but instead of harnessing the power of the Solar wind, ground based lasers would be used to accelerate the craft [5].

The Photon Sail has a very low mass-to-surface ratio of $\sigma_{nom} = \frac{M_s}{A} = 8.6 \times 10^{-4} \text{ kg m}^{-2}$. The Sail can be accelerated to a theorised v_0 of $3.73 \times 10^7 \text{ m s}^{-1}$, or 12.5% of c [5].

By applying equation 4 and 7 once more, we calculated that for the Photon Sail:

$$t_{\frac{1}{2}} = 2.8 \times 10^8 \text{ s} = 8.7 \text{ years.}$$

$$s_{\frac{1}{2}} = 5.1 \times 10^{15} \text{ m} = 0.54 \text{ ly.}$$

Discussion

While the results calculated for Voyager 1 were expected, the relatively small $t_{\frac{1}{2}}$ and $s_{\frac{1}{2}}$ values of the a theoretical Photon Sail were not. If our results are to receive further validation, it will most certainly have a large impact on the possibility of attempting such ambitious missions like Project Starshot [5].

There are a few assumptions made in the calculations that may impact results. The first was that the maximum surface area of the craft was colliding with the atoms. This may impact the results for Voyager 1, however the method in which a Photon Sail is accelerated means that our assumption is likely accurate.

The second assumption we made was that the maximum rate of change of momentum took place during the collisions. For a aerodynamically shaped object, this assumption may be impractical, but both Voyager and the Photon Sail have large surface areas and would act similarly to a parachute, meaning that the real change of momentum would be close to the assumed rate.

Conclusion

We have shown that interstellar hydrogen atoms may significantly decelerate spacecraft with small σ values, such as theorised Photon Sails. We advise that further research focuses on verification of these findings and potential ways to reduce this resistance effect.

References

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