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A1_4 Bending Light With Sound

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Abstract

Sound waves are oscillations of particles in a medium, which means there should be variations in the refractive index of the medium. By treating the waves like gradient-index (GRIN) lenses, we determine whether it would be feasible to use sound waves to produce a visible alteration in light path. We find that a specialised speaker would be necessary, as the minimum displacement amplitude required to achieve a visible effect is $\Delta x = 1.57$ m, with an associated frequency of $f = 6.37$ Hz.

Introduction

Sound waves transfer energy via oscillating the matter in-between two points. Since they physically move the matter they propagate through, there will be periodic changes in the pressure and density of the material; this would change its refractive index.

In this paper, we will attempt to determine what kind of sound will produce visible alteration in light path. We do this by treating the sound wave like a series of gradient-index (GRIN) lenses. GRIN lenses work by utilising a gradually varying refractive index to focus light, instead of geometric curvature.

Theory

The amplitude of the pressure in a sound wave is given by:

$$P_0 = 2\pi f \rho v \Delta x = \omega \rho v \Delta x \tag{1}$$

Where f is the frequency of the sound wave; ω is the angular frequency; Δx is the displacement amplitude (how far the particles move from their mean position); v is the wave velocity and ρ is the density of the medium. To get the dependence of refractive index on pressure, we will use the following relation [1]:

$$n = 1 + \frac{7.9 \times 10^{-4}}{273 + T} P - 1.5 \times 10^{-11} H_r (T^2 + 160) \tag{2}$$

Where, T is temperature in °C, P is pressure in kPa and H_r is relative humidity. Reforming this equation for dry air ($H_r = 0$), in S.I. units:

$$n - 1 = \Delta n(P, T) = \frac{7.9 \times 10^{-7}}{T} P \tag{3}$$

Here, $\Delta n(P, T)$ is the difference in refractive index between the medium and vacuum.

The maximum refractive index variation (Δn_0) will coincide with the maximum pressure, implying that:

$$\Delta n_0 = \frac{7.9 \times 10^{-7}}{T} P_0 = \frac{7.9 \times 10^{-7}}{T} \omega \rho v \Delta x \tag{4}$$

Next, we define our 'visible' condition. The speed of light is much greater than that of sound; therefore, at any moment in time our case is effectively a series of GRIN lenses in two dimensions. Each lens has a refractive index that varies in the form of a cosine peak about the central maximum. GRIN optics [2] tells us that, for a refractive index profile of the form $n(r) = \text{sech}(gr)$, we have:

$$F = \frac{1}{n_0 g \times \sin(gd)} \quad (5)$$

g being the gradient constant; r, the radius from the core (central maximum); d, the length of the 'lens' and F, the focal length.

Approximating our cosine and the sech function to be equal at small r; we can get an effective focal length by saying that $g \approx \frac{2\pi}{4\Delta x}$ and using Eq.(4) to get:

$$F = \frac{2\Delta x}{\pi(1 + \frac{7.9\times10^{-7}}{T} 2\pi f \rho v \Delta x) sin(\frac{\pi}{2\Delta x} d)} \quad (6)$$

Rearranging for f:

$$f = \frac{1.3\times10^6 T}{\pi^2 \rho v F sin(\frac{\pi}{2\Delta x} d)} - \frac{1.3\times10^6 T}{2\pi\rho v \Delta x} \quad (7)$$

Results

Using Eq.(7), we can substitute in relevant values for standard conditions to get a quantitative relationship. Using: $F = 1$ m, $\rho = 1.275$ kgm^{-3}, $v = 330$ ms^{-1}, $T = 297$ K and $d = a\Delta x$, where a will tell us how many times Δx fits in the d (relating the width of the lens to the length):

$$f = \frac{9.30\times10^4}{sin(\frac{\pi a}{2})} - \frac{1.46\times10^5}{\Delta x} \quad (8)$$

Now, if we take a such that it will minimise the frequency needed ($a = 2n - 1$, where n is a positive integer):

$$f = 9.30\times10^4 - \frac{1.46\times10^5}{\Delta x} \quad (9)$$

This is an inverse relationship between frequency and displacement amplitude. Due to the nature of this equation, if Δx is less than $\frac{1.46\times10^5}{9.30\times10^4}$ the frequency is negative, and thus impossible.

To see how plausible this kind of wave is, it is useful to consider the average speed. In one period the wave will have cycled through $4\Delta x$ m in $\frac{1}{f}$ s, meaning that the average speed is:

$$4\Delta x f = v_{avg} = 3.7\times10^5 \Delta x - 5.8\times10^5 \quad (10)$$

Which is a linear relationship, implying that kinetic energy is lower at lower values of Δx. Therefore, the easiest case to achieve is at just above $\Delta x = 1.57$ m with a corresponding frequency of $f = 6.37$ Hz.

Conclusion

The results of this paper show that the displacement amplitude is inversely proportional to the negative of frequency. Also, the optimal/most efficient conditions to achieve our criteria are just above a minimum possible displacement; corresponding to $f = 6.37$ Hz and $\Delta x = 1.57$ m. Furthermore, the width of the sound wave should ideally be of a value $(2n + 1)\Delta x$.

These values would not be possible for normal speakers, which have displacements at much less than a metre. Also, the balance is very precarious as, if you change Δx by a slight amount, the required f changes radically. However, it should be possible, with a very precise speaker, to maintain a specific focal length. The balance is not too important in the context of this paper though, as the change would just indicate a focal length variation; this would still provide visible results, of a mirage-like quality, with a varying optical image.

This result could be used for many applications; for example, the bending of light could be used for cloaking technology, or making lenses out of the air for specific purposes.

References

[1] `http://emtoolbox.nist.gov/Wavelength/Documentation.asp` [Accessed 29 Oct. 2017]

[2] `http://www.grintech.de/gradient-index-optics.html` [Accessed 10 Nov. 2017]