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S1_5 It's a bird! It's a plane! No, it's a pig!

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Abstract

It's likely we have all used the common adynation 'when pigs fly' at some time in our lives, but in this article we question how pigs may fly. Using a Strouhal number of 0.2 we calculated the combined minimum surface area of the wings required for a wild pig to fly to be 5.13 m^2.

Introduction

'When pigs fly' is a hyperbole we are all familiar with, used to convey a complete impossibility when faced with a ridiculous notion. But what if pigs *could* fly? How could they do this? In this paper we investigated the minimum area the wings of a flying pig would need to be to, at the very least, support its weight in flapping flight.

Theory

The minimum size of the wings must support the weight of the pig, i.e. the generated force due to lift must equal the pig's weight. To calculate this force, F_L, we use the lift equation for an aerofoil

$$F_L = \frac{\rho v_x^2 A C_l}{2}, \tag{1}$$

where ρ represents the airflow density, v_x the forward velocity of the body, A the area of the wing and C_l represents the coefficient of lift [1]. C_l collects the complex dependencies of the shape of the body, the inclination to airflow and air viscosity and compressibility. It can be determined by

$$C_l = 2\pi\alpha, \tag{2}$$

where α is the angle of attack. This is the angle of the direction of the aerofoil in relation to the air flow [1].

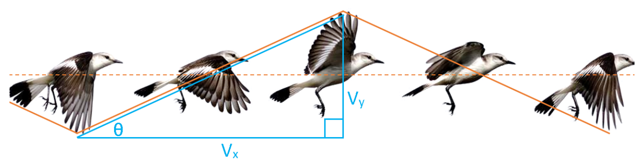

Figure 1: A bird's wings oscillation pattern with constant v_x and v_y at an angle θ [2].

By assuming both the forward velocity during flight and upwards velocity from the wings downstroke, v_y, are constant an expression for the angle θ (see Figure 1) can be found using

$$\tan\theta = \frac{v_y}{v_x}. \tag{3}$$

Considering oscillation amplitude, h and frequency f during flapping flight, the downstroke velocity can be expressed as $v_y = 2hf$.

The dimentionless quantity known as the Strouhal number describes oscillating flow mechanisms and is given by Equation 4. This number represents a measure of the ratio of inertial forces due to unsteadiness of the flow or local acceleration to the inertial forces due to changes in velocity from one point to an other in the flow field [3].

$$S_r = \frac{hf}{v_x}. \tag{4}$$

Equation 3 may therefore be expressed as $\tan\theta = 2S_r$. To simplify the mechanics of flapping flight we assumed a linear relationship between θ and α, such that $\alpha = \theta$ at the wing tip. Disregarding trigonometric identities by use of the small angle approximation, at any distance l along the length of the wing L, α may be expressed as

$$\alpha \approx \frac{2S_r l}{L}. \tag{5}$$

Substituting in $A = Lw$, where w represents the width of the wings and integrating over the length of the wings, the expressions for F_L from Equation 1 can now be rewritten. Equation 6 shows the expression for F_L for both of the pig's wings with Equation 2 and 5 substituted in for C_l and α.

$$F_L = \rho v_x^2 w C_l \int_0^L \mathrm{d}l = \frac{4\pi S_r \rho v_x^2 w}{L} \int_0^L l\,\mathrm{d}l. \tag{6}$$

Finally, the lift force due to downstroke is given by

$$F_L = \frac{4\pi S_r \rho v_x^2 w}{L}\frac{L^2}{2} = 2\pi S_r \rho v_x^2 w L = 2\pi S_r \rho v_x^2 A. \tag{7}$$

Assuming no effects from upstroke, the time averaged lift force $< F_L >$ will therefore be

$$< F_L >= \pi S_r \rho v_x^2 A. \tag{8}$$

This expression can now be equated to the weight of a pig and rearranged for A;

$$A = \frac{mg}{\pi S_r \rho v_x^2}, \tag{9}$$

where m represents the mass of the pig and g represents the acceleration due to gravity.

Results

For the purposes of this article we consider our winged pig to freely soar the skies and hence use the weight and speed of a wild pig. The weight of a wild pig can vary between 79-91 kg for female and male pigs respectively and they may run at speeds up to 30-35 mph [4]. We used the midpoint of both ranges for our calculation such that $m = 85$ kg and $v_x = 14.5288$ ms$^-1$.

The density of air used was that of sea level and at 15°C; $\rho = 1.225$ kgm^{-3} [5]. The Strouhal number for animals falls in the range of 0.2-0.4. Since relatively large birds with flight oscillation amplitude $h > 0.5$ m tend to have a Strouhal number closer to 0.2 [6], we used $S_r = 0.2$.

Computing Equation 9 with these values we find the combined wing area required for a wild pig to fly to be 5.13 m^2.

Conclusion

To conclude, we have calculated the minimum surface area required for a pair of wings to allow a wild pig to fly to be 5.13 m^2. Multiple assumptions were made due to the complexity of the physics of flapping wing flight. However, given the number of simplifications made the accuracy of the value obtained is questionable.

There are a number of factors neglected here that could significantly affect the minimum size required. This would include the frequency of the wingbeat, the strength and mass of the wing material and mass of muscle required etc. Further investigations could account for these factors to improve the reliability of our value.

References

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