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P2_7 The Supersonic Man

J.E.N. Byrne, E.K.F. Lewis, K.A. Dangerfield, S.D. Walton

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

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Abstract

For this paper we performed a simulation in R to calculate the maximum possible speed and mach number of a skydiver jumping from 40,000m and staying in a continuous heads-down position. The parameters are based loosely on the "Red Bull Stratos" project. We found that the skydiver would have a maximum velocity of $444.8ms^{-1}$ at 60 seconds into the dive, reaching mach 1.5, and would stay supersonic for 55 seconds of the flight.

Introduction

On 14th October 2012, Felix Baumgartner performed a skydive from 39,000 metres, reaching a speed of $372ms^{-1}$, and mach 1.24[1]. However, during the jump, he began to spin rapidly, and then spent the remainder of the jump in a spread-eagled position, reducing his maximum theoretical speed [2]. We performed an iterative simulation in R to find out what would have happened if a skydiver was to perform a similar jump in a continuous, stable, head-down position. The simulation gave results for the following parameters: the time taken to reach terminal velocity, the maximum velocity, the peak mach number, and the time the skydiver spends supersonic.

Model

Lerman et. al. [3] describe a model for calculating the velocity of the skydiver as a function of altitude if he were skydiving on Mars, using time as the basis parameter. We adapted this model for use on Earth, changing the temperature, density, and pressure models. Terminal velocity is reached when the force due to gravity and the drag force from air resistance are reached. The

force due to gravity is given by:

$$F_g = \frac{GM_e M_s}{(R_e + h)^2} \quad (1)$$

Where h is the altitude of the skydiver, M_e and M_s are the masses of the Earth and the skydiver respectively, and R_e is the radius of the Earth. The initial value of h used is 40,000m. A value of 118kg [3] was used for the mass of the skydiver. The drag force is:

$$F_d = \frac{1}{2}\rho V^2 C_d A \quad (2)$$

V is the velocity of the skydiver, C_d is the coefficient of drag, taken to be 0.7 [4] throughout the dive, and A is the surface area of the skydiver, taken to be $0.18m^2$ [4] throughout the dive. The density of the atmosphere is a function of both pressure and temperature, and follows the relation:

$$\rho = \frac{P\varpi}{RT} \quad (3)$$

In the equation, ϖ is the molar mass of air, a constant $0.029kgmol^{-1}$ [5]. R is the molar gas

constant. The pressure model we have used for the atmosphere is an exponential decay model based on the formula:

$$P = P_0 e^{-(\rho_0/P_0)gh} \quad (4)$$

P_0 is the atmospheric pressure at sea level (101325 Pa [4]), and ρ_0 is the atmospheric density at sea level (1.2 Kg m^{-3} [4]). For temperature, we used a three layer model given by NASA [6]: below 11km, $T = 288 - 0.0065h$. Between 11km and 25km, $T = 216.5$. Above 25km, $T = 142 + 0.003h$. To find the skydiver's mach number (his velocity as a multiple of the local speed of sound) during the jump, the general formula for the speed of sound in an ideal gas [7] was used:

$$v_{\text{sound}} = \sqrt{\frac{\gamma RT}{\varpi}} \quad (5)$$

Where γ is the adiabatic constant of air, with a constant value of 1.4 [7] used.

Results

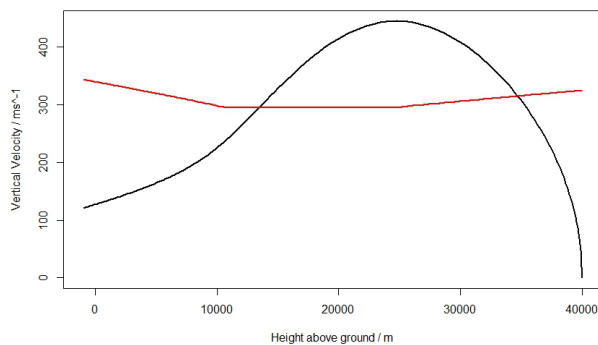


Figure 1: The vertical velocity of the skydiver during the dive against his height above ground (black line), and the speed of sound as it varies with altitude (red line).

Figure 1 shows the results of the simulation. It was found that he reached a maximum velocity of 444.8 ms^{-1} , or 1601 km hr^{-1} at 60 seconds into the dive, when he is 24,800m above the ground. We found that the skydiver becomes supersonic (when his velocity is higher than the local speed of sound, mach one) 35 seconds in, and

becomes subsonic again at 90 seconds into the dive, spending 55 seconds supersonic. The maximum mach number that he reaches is mach 1.5 at 61 seconds into the flight. In altitude terms, the portion where he is supersonic is between 34,700m and 13,200m.

Conclusion

Our model reasonably reproduces the results that were observed in the actual jump performed by Felix Baumgartner, albeit with increased velocity. However, our simulation does show some discrepancies, most visibly the duration of the dive. In his jump, Felix deployed his parachute approximately 1500m above the ground, after 262 seconds of freefall[8]. In our model, it takes the skydiver only 152 seconds to reach 1500m. This may simply be due to our skydiver continually diving head-first, though it may be due to limitations in our atmospheric model. It may also be due to our model not incorporating transonic drag, which can cause a major increase in drag when the skydiver is approaching mach one [9]. Further study would incorporate transonic drag, and a more complex temperature and pressure model.

References

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