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S2_2 Astronomical Tidal Energy

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Abstract

If the Earth ever found itself in the unlikely situation of suddenly orbiting an intermediate black hole of mass 1×10^3 mass of the sun [1], a large problem we would face would be the lack of heat that we originally received from the sun through solar energy. A possible solution for our problem would be to see if the tidal energy produced by the black hole would be enough to match the energy that we would receive from the sun, which is 1.74×10^{17}W [2]. We decided to use an intermediate black hole as an example but the concepts and maths used in this paper should be applicable to any black hole. The output of the tidal energy was calculated to be 1.47×10^7W which is a factor of over a billion times smaller than what we receive from the sun. This would mean that the Earth would be unable to maintain its current surface temperature resulting in life being exposed to a colder and more hostile environment.

Introduction

The orbit of satellites around larger mass objects results in tidal heating being induced in the satellites [3]. We take the unlikely scenario where we suddenly find that the Earth is orbiting an intermediate black hole. We aim to see if the tidal heating induced by the black hole would be of a value where it can match the energy the Earth receives from the sun so that it would be able to sustain the current temperature of the planet.

A calculation of the Earth's new orbital period will be made by making use of Kepler's third law in relation to the intermediate black hole. This will then be used to calculate the power produced by tidal heating. The value calculated will be compared to the 1.74×10^{15}W of solar energy that Earth receives from the sun. Hawking radiation will be ignored as this paper will only focus on the tidal heating aspect that the intermediate black hole will induce.

Equations

The equation that we begin with is Kepler's Third Law:

$$\frac{P^2}{a^3} = \frac{4\pi^2}{G(M+m)} \tag{1}$$

Where P is the period of the orbit of the satellite in seconds, 'a' is the semi-major axis of the satellite in metres, G is the gravitational constant, M and m are the mass of the object being orbited and mass of the satellite respectively in kg [4].

We rearrange equation 1 by multiplying across by 'a' and then square rooting both sides so that we can get the period, P by itself.

$$P = \sqrt{\frac{4\pi^2}{G(M+m)}a^3} \tag{2}$$

$$n = \frac{1}{P} \tag{3}$$

Using equation 2 we obtain the the period which will be plugged into equation 3 to obtain the orbital mean motion of the satellite, n [5].

$$P_T = \frac{21}{2}\frac{K_2}{Q}\frac{(nR)^5}{G}e^3 \tag{4}$$

Equation 4 is the equation used to calculate the energy created through tidal heating [3]. Where P_T is the power produced through tidal heating in Watts, K_2 is the love number which describes the aspect of the rigidity of the satellite, Q is the dissipation of the satellite in question, n being the orbital mean motion of the satellite, R is the radius of the satellite in metres, G being the gravitational constant and e being the eccentricity of the orbit.

Discussion

Equation 1 is rearranged to create equation 2. In equation 2, G is 6.67×10^{-11}m^3 kg^{-1} s^{-2}, m is the mass of the Earth, 5.97×10^{24}kg [6] and 'a' for this case will be 1AU, 1.49×10^{11}m [6]. The mass of M, the intermediate black hole will be 1×10^3 mass of the sun, 1.99×10^{33}kg [1]. Inserted into equation 1 gives us 9.92×10^5s for the period. This time period will be plugged into equation 3 to get the orbital mean motion of Earth which is found to be 1.01×10^{-6}s^{-1}.

In equation 4, the love number and the dissipation both rely on the characteristics of the satellite in question, for this scenario it is Earth and we found that the values 0.308 [7] and 168 [8] are the best to represent Earth the moment it finds itself orbiting the black hole. R is the radius of Earth which is 6.37×10^6m [6]. The value of n will be the number calculated from equation 3. G is the gravitational constant and e for Earth is 0.0167 [6].

The power generated through tidal heating was found to be 1.47×10^7W which is over a factor of a billion smaller than 1.74×10^{17}W which we receive from the sun. This result shows that through tidal heating alone, at the same distance from the black hole as the sun the energy generated will not be enough to maintain global temperature resulting in a colder environment.

Conclusion

With the Earth now orbiting an intermediate black hole, the new period calculated was 9.92×10^5s. This was used to calculate the orbital mean motion which was 1.01×10^{-6}s^{-1}. The output of the tidal energy was calculated to be 1.47×10^7W, compared to the energy produced by the sun which hits the Earth, 1.74×10^{17}W. There was a difference of over a factor of a billion resulting in a lot colder environment. Life on Earth will have harsher living conditions unless another avenue of energy to maintain global temperature is found. A future paper could look at how big a black hole would have to be to heat the Earth up enough to match the sun's energy.

References

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