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# P4_6 Shoot for the Moon

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**Abstract**

In this paper we calculated the maximum mass that could be fired from a 32 MJ railgun to strike the surface of the Moon to be 0.52 kg, assuming an airless Earth. The muzzle velocity of the projectile was found to be 11.1 kms$^{-1}$and the speed at which it would hit the surface of the moon is 1.90 kms$^{-1}$.

## Introduction

In July 2017 the U.S. Office of Naval Research (ONR) announced its new electromagnetic railgun was ready for field demonstrations.[1] Railguns fire projectiles by using electromagnetic forces rather than explosives or propellant as typical ballistic weapons do. This method allows the projectiles to reach very high speeds. In this report we shall examine the scenario of whether the railgun revealed this year is powerful enough to fire a projectile to hit the Moon.

## Theory

Railguns are operated by running a current along a pair of parallel rails with a sliding armature between them. The magnetic field generated by the induced rails accelerates the armature, and thereby the projectile attached to it, to an enormous velocity. The ONR's railgun is currently testing 20 MJ and will be using 32 MJ to launch projectiles by the end of the year. The railgun currently fires BAE Systems' Hyper Velocity Projectile, however, a gun could be created that fires different projectiles with the same energy.[1] To determine the projectile's ability to reach the Moon we shall use the following equations [2]; the first is the kinetic energy equation:

$$E_K = \frac{1}{2}mv^2 \tag{1}$$

The equation for gravitational potential energy:

$$U = -\frac{GMm}{r} \tag{2}$$

And the vis-viva equation:

$$v^2 = GM(\frac{2}{r} - \frac{1}{a}) \tag{3}$$

Where $G$ is the gravitational constant, $M$ is the mass of the body exerting the gravitational force; $m$ is the mass of the projectile; $a$ is the semi-major axis of the orbit; $v$ is the velocity of the projectile and $r$ is the distance from the centre of the body.

## Results

The most efficient orbit to hit the moon is depicted in Fig.1. The gun would shoot at $90^o$ to the surface of the Earth. To find the maximum mass that can be fired to the Moon, assuming an airless Earth, we must first find the energy to reach the equipotential point between the two bodies. This is the point where the gravitational potentials are equal, after this the Moon's gravity will become dominant and pull in the projectile.

Figure 1: The most efficient orbit to reach the Moon.

Using Eq.2 for the Earth and Moon, we can obtain a ratio between the distances from the Earth and Moon. We find that the distance from the equipotential to the centre of the Earth is 81 times the distance from the equipotential to the centre of the Moon. These values are $3.88 \times 10^8$ m and $4.79 \times 10^6$ m respectively, using the semi major axis of the Moon ($3.84 \times 10^8$ m) and the radii of the Earth and Moon: $6.37 \times 10^6$ m and $1.74 \times 10^6$ m respectively. [4] [5]

Using Eq.2, we know that the energy to reach the equipotential is equal to the gravitational potential at the equipotential minus the gravitational potential at the surface, where the railgun is fired. Since the energy that the gun fires at is known (32 MJ), we can rearrange Eq.2 to find the maximum mass that can be fired:

$$m = \frac{E}{(-\frac{GM}{r_{eq}}) - (-\frac{GM}{r_E})} \tag{4}$$

Where $r_{eq}$ is the distance from the centre of the Earth to the equipotential and $r_E$ is the radius of the Earth. This mass is calculated as 0.52 kg. Using Eq.1, the muzzle velocity is therefore 11.1 kms$^{-1}$. We can also find the speed at which the projectile will strike the Moon. First we find the semimajor axis, $a$, of the elliptical orbit, which is the mean of the maximum and minimum of the ellipse: $3.88 \times 10^8$ m (the distance to the equipotential) and $6.37 \times 10^6$ m (the radius of the Earth) respectively. The value of $a$ is therefore $1.97 \times 10^8$ m. Next we use Eq.3 to find the velocity at this point in the orbit, which is 176.8 ms$^{-1}$. Using Eq. 1, where mass is 0.52 kg, the kinetic energy at the equipotential is 8127.1 J. At this point, the bullet has entered the Moon's sphere of influence and the gain in energy can be calculated by finding the difference in gravitational potential energy at the equipotential and the surface of the moon using Eq.2. The potential energy at the equipotential is $5.31 \times 10^5$ J, and at the surface of the moon is $1.47 \times 10^6$ J, so the gain in kinetic energy is $9.34 \times 10^5$ J. Adding this to the kinetic energy of the equipotential and using Eq.1, rearranged for velocity, we find that the 0.52 kg projectile will strike the surface of the Moon with a velocity of 1903.6 ms$^{-1}$.

## Conclusion

In this paper, we have modelled the situation for an airless Earth but in reality it is likely that such a light object would be destroyed by aerodynamic heating as 11 kms$^{-1}$ is the lower limit for meteors to burn up in the Earth's atmosphere. [3] The railgun is being fired at sea level where the air density is much higher. The air around the projectile would ionise due to friction, creating a brief flash of a column of plasma. On an airless body, or those with a tenuous atmosphere, this would not occur. An interplanetary railgun is viable in orbit or on an airless body provided one can generate enough energy.


**References**

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[4] `https://en.wikipedia.org/wiki/Moon`[Accessed 21/10/17]

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