Journal of Physics Special Topics

An undergraduate physics journal

P4_4 Planet Kayakers

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November 2, 2017

Abstract

During this paper we investigate how different surface gravity [1] on Planets within our solar system affects the size and shape of hydraulic jumps in rivers. We find that the ratio of height to length of the hydraulic jump is a constant, to 3 significant figures. This is due to the fact that on planets with low surface gravity the wave is taller but longer and for high surface gravity the wave is lower in height but shorter in length. We conclude that any planet, apart from Jupiter as it violates the initial conditions needed to form a hydraulic jump, would create waves that could be surfed by kayakers and surfers.

Introduction

Kayakers, and some surfers, are always looking for the perfect wave to surf in a river. Waves of such nature are mainly created by a process known as a hydraulic jump. This is produced when an area of fast flowing water meets an area of slower moving water, forming a wave on the boundary line which holds a boat and allows professionals to have fun. If we have colonies living on other planets within the solar system, where will the kayakers call home? We aim to look at how the force of gravity on other planets affects the size and shape of such waves.

Theory

A hydraulic jump is a form of standing wave created in rivers when an area of supercritical flow meets an area of sub-critical flow. For supercritical flow the water is moving faster than the upstream propagation speed of any disturbances, meaning that features upstream cannot be changed by features downstream. The opposite effect occurs for sub-critical flow, features downstream can alter the flow of water upstream [2]. One prime example of a hydraulic jump is the wave that is formed after a weir on a water way. The change in height from smooth concrete to rough river bed alters the Froude number [3] forming the rolling white wave that can be seen. For a visual example we recommend viewing the image found via the fourth reference [4].

The Froude number determines the ratio of gravitational to inertial forces and is the main characteristic in whether the flow is supercritical or sub-critical. If the Froude number is greater than 1 the flow is supercritical and if it is found to be lower than 1 it is sub-critical.

Equation 1 [5] is used to find the Froude number where v is the velocity of the flowing water, g is the surface gravity of the body and y_1 is the height of the water above the hydraulic jump (upstream). The Froude number is important as it is used in calculating both the height and length of the hydraulic jump. We created a hypothetical situation where water pumps create an artificial course in a dome pressurised to

Table 1: A table showing the results calculated using Equations 1, 2 and 3

Planet	Surface Gravity (ms^{-2})	Froude Number	Froude Number Squared	Height (m)	Length (m)	Height/Length Ratio
Mercury	3.70	2.33	5.45	2.76	199.99	0.14
Venus	8.87	1.51	2.27	1.04	7.62	0.14
Earth	9.81	1.43	2.06	0.88	6.51	0.14
Mars	3.71	2.33	5.44	2.75	19.95	0.14
Jupiter	24.92	0.90	0.81	-0.20	-1.51	N/A
Saturn	10.44	1.39	1.93	0.79	5.85	0.14
Uranus	8.87	1.51	2.27	1.04	7.62	0.14
Neptune	11.15	1.34	1.81	0.70	5.17	0.14

1 atmosphere. The velocity and depth of the water above the hydraulic jump is engineered to be $5.5ms^{-1}$ and $1.5m$ respectively. These values were chosen as they create a hydraulic jump close to the ones found at white water centres.

$$F_r = \frac{v}{\sqrt{gy_1}} \tag{1}$$

The height of the jump is calculated by Equation 2 [5], where the variables have the same meaning as above.

$$H_j = \frac{y_1\sqrt{1+8F_r^2} - 3y_1}{2} \tag{2}$$

The length of the hydraulic jump can be calculated by Equation 3 [5]. This equation was derived from experimental methods and observing hydraulic jumps that were created in labs.

$$L = 220 \times y_1 \times tanh(\frac{F_r - 1}{22}) \tag{3}$$

Discussion

The results in table 1 show every planet apart from Jupiter will create a hydraulic jump. This is because the surface gravity [1] on Jupiter is so high that the initial Froude number is less than one, meaning the upstream flow is sub-critical and hydraulic jumps can only be formed when the flow of water changes from supercritical to sub-critical. As the flow is initially sub-critical, the flow is already turbulent, and adding a feature to create a hydraulic jump would not have the desired effect.

Conclusion

If humans were ever to colonize planets within our solar system, kayakers can paddle on 7 of the 8 planets. Each planet has its pros and cons; one with low surface gravity will create small waves but will allow huge jumps to be performed. However planets with high surface gravity will create enormous waves but they may be too large and the gravity will be too high to allow any stunts to be performed.

References

[1] http://www.smartconversion.com/otherInfo/gravity_of_planets_and_the_sun.aspx [Accessed 17 October 2017]

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[5] https://en.wikipedia.org/wiki/Hydraulic_jumps_in_rectangular_channels [Accessed 17 October 2017]