Journal of Physics Special Topics
An undergraduate physics journal

A2_4 Moon Migration Weighing You Down?

R. Bradley-Evans, R. Gafur, R. Heath, M. Hough

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

November 2, 2017

Abstract

It is theorized the Moon formed approximately 60 Myr after the formation of the Earth during a globally-cataclysmic event widely known as the Giant Impact Hypothesis. Since its birth, the Moon's distance from Earth has gradually increased as a result of tidal forces. We explore the effect this would have on a 60 kg_F human on Earth and find a difference of 0.34 kg_F were they to be present across the Moon's history. For the sensation of weightlessness, the Moon would need to orbit within the surface of the Earth.

Introduction

The Moon is thought to have formed out of a two-body collision between Earth and a Mars-sized planetesimal 4.53 Gyr ago - known as the "Giant Impact Hypothesis" [1]. Subsequently, as the Moon's fragments coalesced, a tidal interaction with the Earth began. This resulted in a slow, outwardly migrating orbit causing a lengthening of the Earth's rotation period due to the conservation of momentum. As the Moon continues to migrate, this effect will ultimately dissipate as the tidal forces weaken. We explore the effect this migration would have on a human being if they were to be present on the Earth's surface throughout the lifetime of the Moon.

Theory

Using Sharma et al. equation for the Moon's orbital radius as a function of time [1] with Newton's Law of Universal Gravitation [2], we derived a comparison between a human's present weight on Earth compared with that over the course of the Moon's history. This difference exists due to the downward and upward gravitational forces exerted by both the Earth and Moon respectively with respect to the surface of the Earth. In the present day, as a result of greater proximity and mass, the Earth's gravity dominates over the Moon's influence.

This motivated us to explore the effects of a stronger gravitational force from the point of formation and how a human's weight would differ compared with the present. For clarity, we use the units of "$Kilogram - Force$" (kg_F) in our findings (henceforth abbreviated as KF). This unit is equal to the force due to 1 kg of mass at the surface of Earth where the gravitational acceleration is 9.81 ms^{-2}, and can therefore be treated as a weighted force specifically for the Earth environment. Although not an SI unit, it aids in the understanding of the results whilst preserving the physical distinction between "$Mass$" and "$Weight$", and circumvents the difficulty in comparing values in units of Newtons.

Results

For the change in Lunar orbital radius, Sharma et al. devised an equation through trial and error to approximate the outward migration of the Moon's orbit, which is given by

$$\begin{aligned} r_l = D - (D - E)\exp\left[-t/t_1\right] \\ -F\exp\left[-t/t_2\right] \\ +F\exp\left[-t/t_3\right], \end{aligned} \quad (1)$$

where $D = 5.52887891 \times 10^8$ m, $E = 0.157 \times 10^8$ m, $F = 1.6 \times 10^8$ m, $t_1 = 2$ Gyr, $t_2 = 14$ Gyr, and $t_3 = 1.15$ Gyr are all parameters which, together, model the migration of the Moon. t is the time since formation in Gyr [1]. We then derived an expression for the net gravitational force, provided we assume the calculations are completed as the Moon is overhead and the system's mass has remained constant, which was found to be

$$\Delta F_G = Gm\Big[\frac{M_E}{R_E^2} - \frac{M_M}{(r_l - R_E)^2}\Big], \quad (2)$$

where G is the gravitational constant (6.67×10^{-11} $m^3kg^{-1}s^{-2}$), m is the human's mass (taken to be 60 kg), M_E and M_M are the Earth's and Moon's mass respectively (5.972×10^{24} kg and 7.346×10^{22} kg), and R_E is the Earth's radius (6371 km) [3]. As this form was not useful in comparing with the mass of the human at present, Eq. (2) (in units of N) was converted to kg_F by dividing through by the current acceleration due to gravity, giving

$$\begin{aligned} \Delta W &= \frac{\Delta F_G R_E^2}{GM_E} \\ &= m\Big[1 - \frac{M_M R_E^2}{M_E(r_l - R_E)^2}\Big]. \end{aligned} \quad (3)$$

Plotting Eq. (3) against the logarithm of time (measured in Gyr) yields Fig. 1, showing a difference of 0.34 kg_F when the Moon was significantly closer to the Earth 0-500 Myr ($r_l = 0.16 - 0.84 \times 10^8$ m) after formation. Using the same method, for a human to feel weightless (0 kg_F), the Moon would need to orbit at ~ 7100 km, placing the Moon ~ 700 km above the Earth's surface and thus it's surface ~ 1000 km within the Earth's ($R_M = 1738$ km) [3].

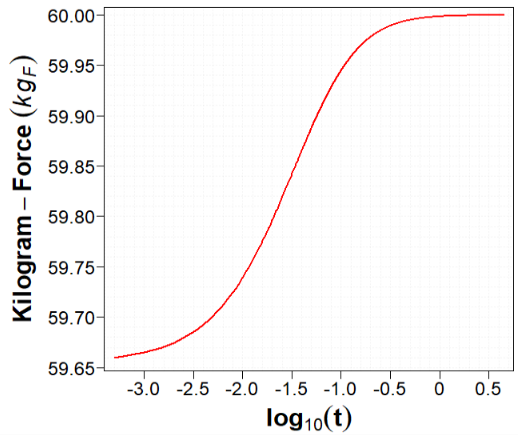

Figure 1: Varying KF across the Moon's lifetime, showing a steep gradient associated with the stronger gravitational presence at shorter distances to Earth.

Conclusion

We find that for the majority of the Moon's history, the gravitational effect on a human's weight would be marginal; only 0-500 Myr from formation caused any noticeable deviation with a maximum difference of 0.34 kg_F. The results also suggest that the sensation of weightlessness would arise if the Moon could orbit at ~ 7100 km, placing the Moon within the surface of Earth. This, however, requires further work to calculate the gravitational effects of non-point masses at very short distances. Our findings encapsulate the weak strength of the gravitational force resulting from its dependence on the inverse-square of the distance between the centres of two (or more) masses. Further to this, it shows that in a practical sense, the Earth's gravity dominates when considering surface masses.

References

[1] B.K. Sharma, B. Ishwar, N. Rangesh, *Simulation Software for the Spiral Trajectory of our Moon* (Advances in Space Research, 2009), Vol. 43, p. 460-466

[2] P. A. Tipler, G. Mosca, *Physics: For Scientists and Engineers* (2008)

[3] `https://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html` [Accessed 10 October 2017]