Journal of Physics Special Topics

An undergraduate physics journal

A3_1 Simple Harmonic Rex

D. Cooper, S. Reiness, M. Sidhu and C. Watson

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

November 15, 2017

Abstract

Artistic licence often tests fundamental physical laws in contemporary cinema, allowing for unrealistic yet creative scenes. Nevertheless, these improbable scenarios can be valuable exercises in applying our physical knowledge to unique situations. From the 1999 movie 'Toy Story 2', using simple harmonic motion, we calculated the spring constant of Slinky the Dog's spring and the friction coefficient between his paws and the rooftop he was standing on. The value for the spring constant was found to vary between 9.04 Nm^{-1} and 2.63 Nm^{-1} and the friction coefficient was found to vary between 1.69 and 4.90. These are then compared to typically used values to assess the scenario's plausibility.

Introduction

In the 1999 sequel 'Toy Story 2', Slinky (Slink) is seen being used as a bungee chord by the other characters. Due to Rex the Dinosaur's nervous disposition, a brief moment of simple harmonic motion (SHM) is seen whereby Rex oscillates on the tail-end of Slink. From measuring the period of the oscillations, we calculated the spring constant of the slinky connecting Slink's two halves. As Slink's upper half does not seem to move during this period of oscillation, the coefficient of static friction of his paws was also calculated. Furthermore, during periods of oscillation without Rex or other characters attached, a verifying spring constant was calculated and compared to that derived from the motion with Rex attached. Assumptions made for this analysis were that: the slinky spring is massless, there is no energy lost to surroundings and the mass of Slink is distributed evenly over his two halves.

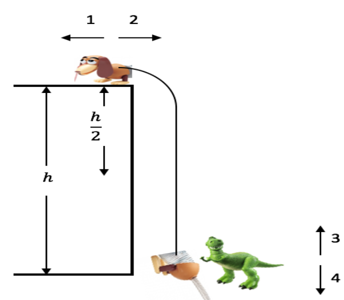

Figure 1: Exhibiting the geometry of the problem, from point $t = 0$ seconds. Numbers denote forces: 1= friction from the paws, 3 = restoring force from the spring, 4 = the weight of the tail and Rex and 2 = the resolved force of 3 and 4.

Spring Constant

The basis of all calculations for the spring constant of Slink, k, stems from the fact that the Rex-and-tail system are undergoing SHM. This allows Hooke's law to be equated to the result of Newton's second law to give

$$m\ddot{x} = -kx, \tag{1}$$

where m is the mass on the end of the spring, $\ddot{x}$ is the mass' acceleration and x is the mass' displacement from its equilibrium position; as Rex always returns back to the height of the roof, this is half way up the building. Eq. (1) is a differential equation, easily solved by inspection. The solution is $x = A\cos\left(\frac{2\pi}{T}t\right)$ where A is the amplitude of the oscillations ($\frac{h}{2}$, see Fig. (1)), T is the period of an oscillation and t is time. This solution is valid if the constants present in Eq. (1) are related by

$$m\left(\frac{2\pi}{T}\right)^2 = k. \qquad (2)$$

Fitting Eq. (2) to our scenario, $m = \frac{m_S}{2} + m_R$ where m_S is the mass of Slink and m_R is the mass of Rex.

Using Eq. (2) with $m_S = 0.599$ kg [1] and $m_R = 0.998$ kg [2], k values for different scenarios were calculated.

Scenario	T (s)	k (Nm^{-1})
With Rex	2.38	9.04
Without Rex	2.12	2.63

Table 1: Showing the measured values of oscillation period and values of the spring constant.

Coefficient of Friction

The frictional force and the coefficient of static friction are related by

$$f_s \leq \mu_s F_n, \qquad (3)$$

where f_s is the frictional force, μ_s is the coefficient of static friction and F_n is the normal force. In this instance, F_n is equal to the weight of the head end of Slink ($\frac{m_S}{2}$). As his upper half remains stationary and his connection to the spring is approximately perpendicular, we assumed that the vertical forces are translated into horizontal forces. Thus, f_s is supplied by the net vertical force. Rearranging for μ_s and making substitutions for f_{max} and F_n gave

$$\mu_s \geq \frac{kx - g(\frac{m_S}{2} + m_R)}{\frac{m_S}{2}g}. \qquad (4)$$

With all relevant values inserted into Eq. (4) and $x = h/2$ (h of average garage is $\approx$ 3m), the values for μ_s are $\mu_s \geq 4.90$ or $\mu_s \geq 1.69$ with or without Rex attached, respectively.

Conclusion

As seen above, values for k and μ_s for Slink appear to vary depending on the situation. Also, our calculated value of k does not match that of real life slinky toys; typically k less than 1 Nm^{-1} [3], differing to our results by a factor of at least 9.04. Moreover, the value for μ_s is much larger than those seen in nature. Of course, assumptions made in this paper may induce errors that taint our calculated result.

Given our results and their contrast to typical values, this scene does not seem a likely occurrence in the natural world, nor is it consistent with itself. Instead of an oscillation with an amplitude the size of half the height of a garage roof, the slinky would recoil back but only by a very small amount, given its real value of k; this is arguably the biggest cause of error in the entire investigation. It appears a majority of the errors are supplied by the animation itself, unfortunately deeming this scene a mere piece of 'Hollywood magic'.

References

[1] `https://www.amazon.co.uk/Official-Disney-Talking-Slinky-Figure/dp/B013K50YBG/ref=pd_sim_21_2?_encoding=UTF8&psc=1&refRID=YJ5KBBG0HHSV9PFVZXQD` [Accessed 8 October 2017]

[2] `https://www.amazon.co.uk/Disney-Collection-Talking-Dinosaur-Phrases/dp/B00B9C911W/ref=pd_bxgy_21_img_2?_encoding=UTF8&psc=1&refRID=4GME508HPY5PTVSRV5V8` [Accessed 8 October 2017]

[3] R. C. Cross and M. S. Wheatland *Modelling a Falling Slinky*, American Journal Of Physics, Vol. 80, 1051 (2012)