P4_1 Somewhere over the rainbows!

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October 23, 2017

Abstract

In this paper, we have discussed what a rainbow produced by dispersion through water droplets looks like from the surface of a planet orbiting a binary star system. The physics of rainbows was examined and the radius of curvature was found to be 42°. How rainbows would appear on a real planet orbiting a binary star system, Kepler-34b, was studied and we found that two rainbows, 12° apart and 84° across, would be visible on the planet's surface.

Introduction

Circumbinary planets, that is a planet orbiting a pair of stars that orbit each other, are a staple of science fiction and the galaxy. On Earth we see the phenomena of rainbows as light from the sun is dispersed by falling rain droplets, but how would rainbows appear on a planet with two or more suns?

Theory

Rainbows occur on Earth when light is dispersed by thousands of water droplets, usually from rain. Several rainbows are produced, each of them a different order determined by the number of reflections the light makes inside the droplet. Each order is fainter than the last so typically only the first order is seen, although technically every rainbow is a "double" rainbow.

Using Figure 1, we can see that light of a first order rainbow (A.) enters the droplet and is refracted at the air-water interface (B.); reflected at the water-air interface (C.) and finally refracted at the air-water interface (D.) As the index of refraction of a medium depends slightly on the wavelength of the light [3], with the red
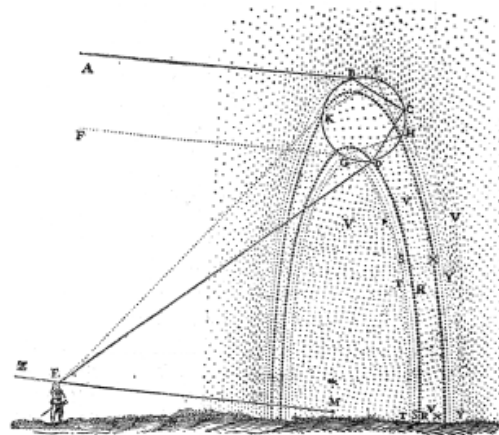

Figure 1: How first and second order rainbows are made. By René Descartes.[1]

part of the spectrum at a slightly larger angular radius, the rainbows appears as a continuous band of colour with red at the outer side.

The angular radius of curvature of a rainbow can found using Snells law:

$$n_1 sin(2\beta - \varphi) = n_2 \sin(\beta) \tag{1}$$

where 2β is the angle of internal reflection, the angle of incidence of the sun's rays with respect

to the drop's surface normal is $2\beta - \varphi$, and n_1 and n_2 are the refractive indexes of air and water at 1 and 1.333 [4] respectively.

Rearranging Eq.(1) for φ gives,

$$\varphi = 2\beta - \arcsin(n \sin \beta) \qquad (2)$$

where 2φ is the angular radius of curvature. The rainbow will appear when the angle φ is maximum with respect to the angle β. Setting $d\varphi/d\beta = 0$ we find $\beta_{max} \approx 40.02^o$. Substituting this value into Eq. (1) we obtain the angular radius of a rainbow of 42°. [2] Regardless of where one stands, the size of a rainbow always appears the same and is on the opposite side of the sky from the sun.

Circumbinary planets have a critical radius from the stars that they must exceed to be in a stable orbit. This critical radius varies from 4 to 8 times the separation of the stars. [5] We shall use an imagined moon with liquid water orbiting the gas giant Kepler-34b in the system Kepler-34 as it contains stars like our sun with very similar brightness to each other. This means rainbows from from both stars would be visible at the same time. With a separation between the stars of 0.2318 AU and Kepler-34b's semi-major axis 1.090 AU we can use trigonometry to find the angular separation from the perspective of the planet. [6]

$$\tan^{-1}(\frac{0.2318}{1.090}) \qquad (3)$$

From this we calculate a maximum separation between the stars of 12.00^o.

Discussion

As the full arc of the rainbow is 84° across and 42° above the horizon due to the water the light is dispersing through, the star's characteristics only affect the brightness and position of the rainbow. From this we can conclude that the rainbows from stars in a circumbinary system would always overlap as the stars could not appear separated by more than 84° as the critical radius is proportional to the stars' separation. In the case of our example system Kepler-34 the

rainbows, at the point where the stars appear the furthest apart in the sky, would be equal in radius, 42°, and the top of the arcs 12° apart. The bows would overlap and the colour would double in brightness as double the amount of light would enter your eye.

Conclusion

Overall, rainbows on circumbinary planets around two stars of similar brightness would appear as two rainbows, with exactly the same dimensions, overlapping. Planets with one star much brighter than the other would be very unlikely to see the rainbow from the dimmer star because the rainbow would be too faint. In a system where planets orbit more than two stars you would see as many rainbows as there were stars in the sky, provided the stars where all of similar brightness. Overlapping rainbows would definitely be a sight to behold.

References

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[6] http://www.openexoplanetcatalogue.com/planet/Kepler-34%20%28AB%29%20b/ [Accessed 15/10/17]