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P2_13 Dominoes and Self-Organised Criticality

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Abstract

This article presents a very simple model of self-organised criticality using a 1 dimensional system of dominoes. It is found that when the likelihood of a domino falling is small, a power law exists between number of dominoes which fall in each event, x, and the frequency with which it occurs, y. This is determined to be of the form $y=x^{-1.88}$.

P2_2 Complex Systems

Introduction

Self-organised criticality is a phenomenon where a dynamical system arranges itself into critical state where it exhibits different properties from its equilibrium state [1]. Some systems have a critical state such as a triple point, where solid, liquid and gas exist together at a critical temperature and pressure. However, the importance of self-organised criticality is that systems with this properly naturally arrive at their critical point; it is an "attractor". This article describes a simple system which exhibits self-organised criticality with the benefit of being non-abstract, such that with enough time it could be verified experimentally.

Model

Consider a scientist who finds himself in a room with nothing but a large box of dominoes. Sooner or later, he is bound to succumb to his nature and begin experimenting. He decides to set up a circle of n dominoes, spaced such that the distance between dominoes is just under the length of a domino.

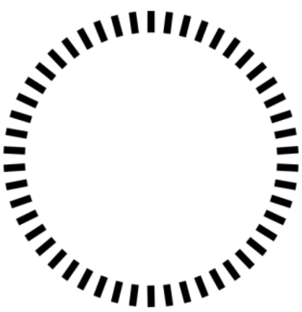


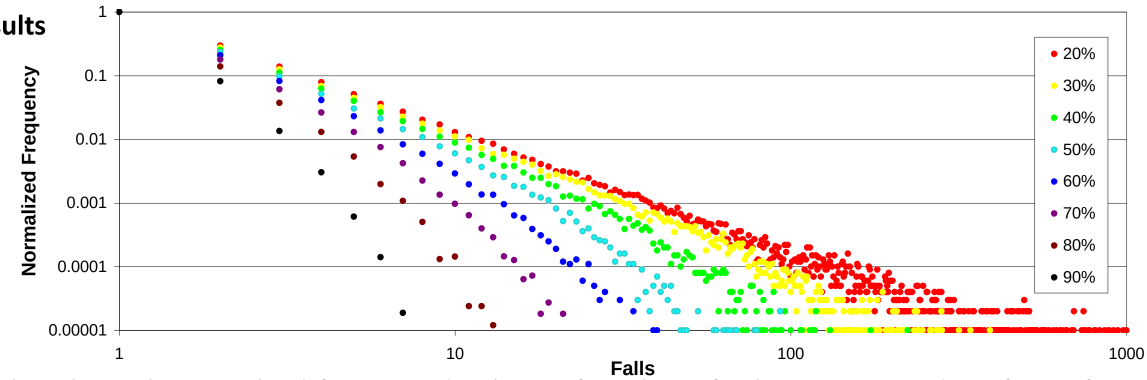
Figure 1 shows how we arrange a circle of 50 dominoes.

The exercise is complicated by the fact that our scientist, jittery from over-excitement, has a certain probability p that upon placing the next domino he will fumble, and knock it over. When this happens, it will fall forwards or backwards with equal probability. Since they are placed sufficiently close, if a domino falls towards an adjacent domino that too will fall over and a chain reaction occurs until a domino falls into a space. Exasperated but determined, the scientist removes any dominoes that fall and continues.

Figure 2 shows how fumbling one domino affects a line of dominoes in one direction until there is a space.

This experiment is carried out using NetLogo for a circle of 1,000 dominoes. A circle of empty cells is created, marking sites where dominoes can be placed. Dominoes are added to random empty cells in the circle one at a time. Each time, the domino being placed will fall with probability p. If it does fall, it falls clockwise or anti-clockwise with equal probability. If a domino falls into an adjacent domino, that domino falls and the chain reaction continues until a domino falls into a space.

Results



Graph 1 shows the normalized frequency distribution for values of p between 0.2 and 0.9 for n of 1000, plotted on a logarithmic scale. Each data set is of length 110,000 to 180,000 total falls.

Discussion

Graph 1 shows that for high values of p, the trend on a log-log plot is a curve downwards, becoming steeper with more falls. This is characteristic of an exponential function and is somewhat uninteresting. An exponential falls off by the same fraction with each extra domino in the fall. Clearly then it is simply the product of probabilities as we might expect.

However, when the scientist has gathered his wits, and reduced p to 0.3 and 0.2, the log-log plot, shows the yellow and red points form a clear straight line. Collecting more and more data would be necessary to smooth this out, but would only be practical with a very powerful computer. A similar result is found in the Bak-Chen-Tang forest-fire model [2], where critical behaviour exists only when fires (analogous to fumbling a domino) are infrequent.

On a log-log plot, a straight line indicates the data follows a power law. In this model, the scientist drives the system by adding dominoes. With few dominoes, large cascades are unlikely and the number of dominoes increases. Above a certain point, large sets of dominos become likely to fall, reducing the dominoes in the system. On the threshold between these states is the critical point. The emergent power law is a product of this state.

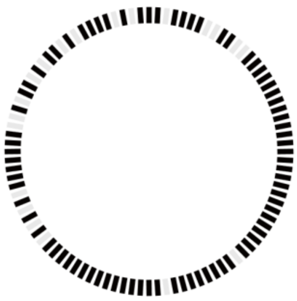


Figure 3 shows a ring of 100 dominoes organised into a critical state. One wrong move and a wide range of cascades can occur.

Of the features of a power law [3], the most interesting here is its scale invariance. If you imagine existing as a point on a power function, it would be impossible to determine what scale you are on as throughout the function, the same is always true; if you double the x co-ordinate, the y value decreases by a constant factor.

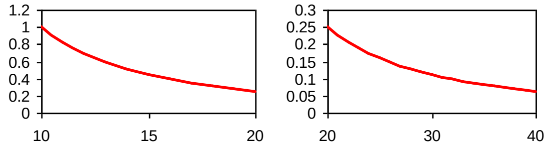


Figure 4 shows a plot of the function $y=100x^{-2}$. Notice that without the scales, they would be indistinguishable.

Conclusion

In this domino model, the frequency plot tends towards the function $y=x^{-1.88}$ where y denotes the normalised frequency and x is the size of the falls. Therefore a cascade of twice the size of another always occurs $2^{1.88}$ times less frequently. With low enough p, the scientist can eventually complete his task (technically, with any $p < 1$ it can be completed, though can become very unlikely). It would certainly be worth looking at the distribution of times taken over a range of p.

References

[1] P. Bak, C. Tang, K. Wiesenfeld, (1987). *Self-organized criticality: an explanation of 1 / f noise. Physical Review Letters* **59**: 381–384

[2] P. Bak, K. Chen, C. Tang, (1990), *A forest-fire model and some thoughts on turbulence. Phys. Lett. A* **147**, 297–300.

[3] M. Newman, (2005), *Power laws, Pareto distributions and Zipf's law. Contemporary Physics* **46**: 323–325