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P2_14 Shotgun patterns

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Abstract
This article provides a model of particle collisions for producing shot patterns from shotgun ammunition of a constant mass, for different numbers of pellets. The scatter pattern is found to be Gaussian where, interestingly, the standard deviation is slightly greater for fewer large pellets, than the same mass of smaller pellets over all ranges.

P2_2 Firearms

Introduction
Modern shotguns are able to chamber a wide variety of ammunition types, though they all use the same core mechanic. The cartridge contains a number of spherical pellets known as the shot. These are accelerated out of the barrel by an explosive charge contained in the cartridge.

Having left the shotgun, the shot is no-longer bounded by the barrel, and the pellets begin to disperse. This is caused by a small distribution of velocities tangential to the direction of the shot. Collisions between pellets cause the bulk to disperse over time.

The paper aims to determine degree of this dispersion as a function of distance, and find how this changes for different numbers and sizes of pellets.

Model
To produce a simple agent-based model in NetLogo, the tightly packed cluster of pellets is taken at the point they leave the barrel. The model will begin by creating the shot. To create different types of ammunition, we consider *n* pellets of total mass *M*. To simplify the situation further, the shot will be arranged as a single layer in a square lattice, and disperse in 2 dimensions only. This is illustrated in figure 1.

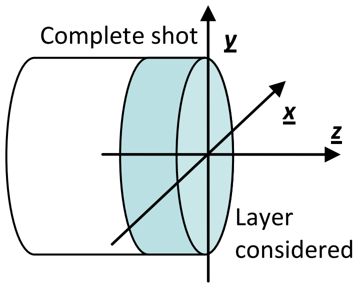


Figure 1 shows a single layer of pellets, with dispersion occurring in the $\underline{x}$-$\underline{y}$ plane.

The next step is to create a model in which the individual pellets collide realistically. For small particles, these equations are simple, though here we have collisions between relatively massive particles. Since modern shot is made from steel, these collisions can be taken to be elastic. When 2 pellets, labelled 1 and 2, come into contact, we begin by defining the vector $\underline{\hat{n}}$. as the unit vector pointing from the centre of 1 to the centre of 2. Another vector, $\underline{\hat{t}}$ is the unit tangent to both pellets. These are shown graphically in figure 2.

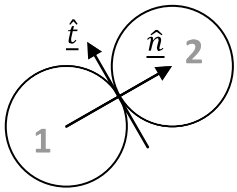


Figure 2 illustrates the geometry of a collision between 2 pellets.

These vectors are important, as the only changes in velocities that occur are in the direction of $\underline{\hat{n}}$. The scalar normal and tangential velocities of the particles are given simply by

$$v_t = \underline{v} \cdot \underline{\hat{t}} \qquad v_n = \underline{v} \cdot \underline{\hat{n}}$$

where *v* is the pellet velocity, and a subscript *n* or *t* indicate the normal or tangential components of velocity respectively. Since only the normal component is changed, the scalar tangential velocities post collision, v_t', are simply given by

$$v_t' = v_t$$

The scalar normal velocities can be found using the equations for a 1-dimensional collision [1] in the $\hat{n}$ direction. For particle 1,

$$v_{1n}' = \frac{v_{1n}(m_1 - m_2) + 2m_2 v_{2n}}{m_1 + m_2}$$

Since the pellets have equal masses, m_1 and m_2 are the same, so this conveniently simplifies to

$$v_{1n}' = v_{2n}$$

Finally, scalar velocities v_{1t}' and v_{1n}' are translated back into vector form by multiplying by the unit tangent and unity normal respectively, and finally combined to give the post-collision velocity v1'

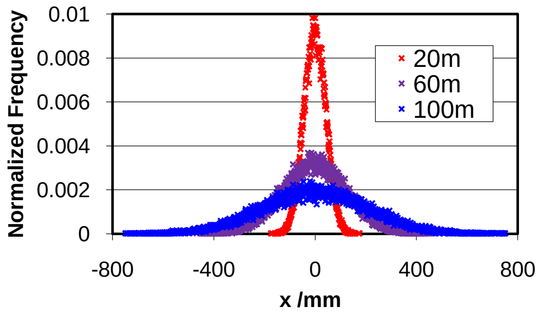
$$\underline{v_1'} = v_{1n}'\underline{\hat{n}} + v_{1t}'\underline{\hat{t}}$$

Note that the method for finding the velocity of particle 2 is exactly the same, except with each subscript 1 and 2 reversed.

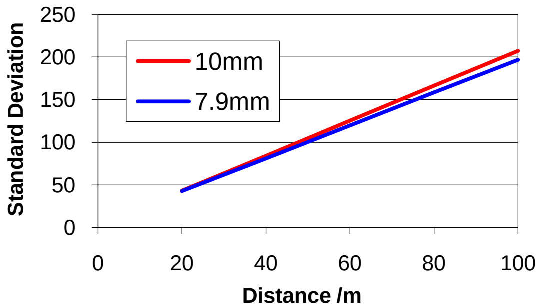
Upon leaving the barrel, pellets will not collide at all if they only have velocity in the z direction. Therefore they are taken to have a small Gaussian distribution of velocities in the ***x*** and ***y*** directions, though the vast majority of their energy is in the shot direction, ***z***. Velocity in ***z*** is taken as 500ms^{-1} [2], and velocities in ***x*** and ***y*** are given by a normal distribution with a mean of 0 and a standard deviation of 0.5% of velocity in ***z***. The 0.5% here is arbitrary for the model. It could be tuned to more closely reflect the real case if data was available to tune the model to.

Since this is a 2-dimensionsal model, it is useful as a way of investigating the differences between large numbers of small pellets, and fewer larger pellets. The model is run for 60,000 pellets in 2 different arrangements; 6 pellets and 12 pellets with a ratio of sizes 10mm:7.9mm such that the total volume, and therefore mass of pellets is the same in both cases. Similarly, the drag on pellets is neglected, so that results reflect only the outcomes of pellet collisions. In each run, the displacement of pellets from the centre is measured at 20m intervals out to a range of 100m.

Results



Graph 1 shows the pellet distribution at 20, 60 and 100m for a shot of 6 pellets of 10mm diameter.



Graph 2 shows the standard deviation from the mean of pellet distributions with range for 6 10mm pellets and 12 7.9mm pellets.

Conclusion

Graph 1 clearly shows Gaussian distributions of pellets across all ranges, becoming flatter and wider with increasing range. Graph 2 shows how the standard deviation of these distributions increases linearly with range.

Although it may seem counter-intuitive, the model shows fewer larger pellets have a slightly greater spread than a larger number of small pellets. This may be because the increased number of collisions initially homogenises the distribution of velocities between pellets.

With the inclusion of drag forces, a larger number of pellets will decelerate at a higher rate. The result of this would be that smaller pellets would appear to spread out more than larger ones over the same range. The model demonstrates this is more significantly related to their velocity in ***z***, not dispersion in ***x***-***y***.

References

[1] One dimensional collisions, *www.nvcc.edu/home/nvmajew/phy241/labs/lab6.html*

[2] The Shotgun, *www.angelfire.com/tx/ShotGun*