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A2_4 An (almost) ring of light

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Abstract

A potential method for causing light to travel a quasi-circular path by mimicking the shape of an N-sided polygon through a series of refraction events is discussed. Some simple geometric calculations are made in order to learn how this may be achieved and forms of intensity loss are considered. It is concluded that further analysis is required as there appears to be a trade-off between the beam attenuating too much and the refractive indices required being unphysical.

Introduction

Fermat's principle dictates that a beam of light travelling from one point to another will always take the path that can be traversed in the least time i.e. a straight line. However, a series of straight lines – each with a small, regular external angle between them – would be indistinguishable from a curved path. This 'bending' of light could be achieved through the use of a series of transparent blocks with a steadily increasing refractive index.

Discussion

In order to achieve a quasi-circular path for the beam, first a regular polygon of a sufficient number of sides must be chosen for the light-path to mimic. The interior angle of this shape in degrees, α, can be calculated using,

$$\alpha = \frac{180(N-2)}{N}, \tag{1}$$

where N is the number of sides of the chosen polygon [1].

Figure 1 shows a theorised set up to achieve a curved beam. If the internal angle between the incident beam and the refracted beam is chosen to be α, the desired angle of refraction, θ_r can be calculated using simple geometry (Eq.2), and therefore the refractive index necessary to achieve this can be calculated using Snell's Law (Eq.3).

$$\theta_r = \alpha + \theta_i - 180, \tag{2}$$

$$n_2 = n_1 \frac{\sin\theta_i}{\sin\theta_r}, \tag{3}$$

where θ_i is the incident angle of the beam, n_1 is the refractive index of the initial medium and n_2 is the refractive index of the first block.

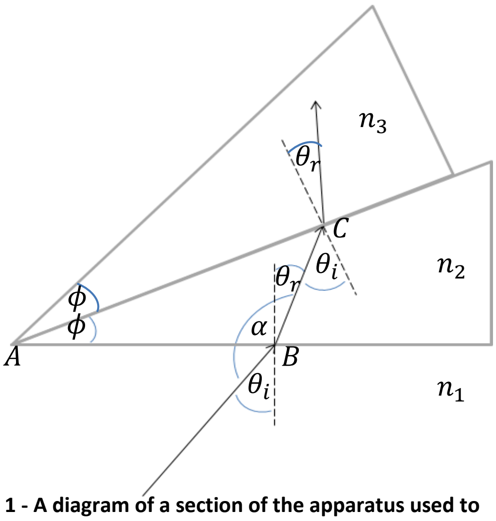

Figure 1 - A diagram of a section of the apparatus used to achieve a circular beam. The initial beam, and its path through the first two blocks is depicted with the relevant angles. $n_3 > n_2 > n_1$.

However, there is one more angle to be considered. In the theorised set up, the calculations can be simplified greatly if the angle of incidence into each new medium is the same (θ_i is constant). Therefore, the type of block used must be triangular in shape in order to accommodate for this 'adjustment angle', ϕ. Again, simple geometry allows for calculation of ϕ using the two known angles of the triangle ABC, as shown in Eq.4,

$$\begin{aligned} \phi &= 180 - ABC - BCA \\ &= 180 - (90 + \theta_r) - (90 - \theta_i) \\ &= \theta_i - \theta_r \,. \end{aligned} \tag{4}$$

Using this angle ϕ, each subsequent medium can have the same dimensions; after the beam is refracted, it will be incident to the next block at the same θ_i as it was to the previous block prior to refraction (it should be noted that while ϕ is not used in later calculations, it is necessary for the construction of such a device). The advantage of having θ_i be constant is that the result of Eq.2 will also be constant. With a rearrangement of Eq.3, it can be seen that constant values of $\theta_i$and θ_r means that the ratio of the refractive indices can be expressed as a constant, C,

$$\frac{n_2}{n_1} = \frac{\sin\theta_i}{\sin\theta_r} = C. \quad \textbf{(5)}$$

Via induction, and assuming the initial medium to be air ($n_1 \approx 1$), it can be said that the necessary refractive index for any subsequent media can be found using

$$n_m = C^m, \qquad m = 1, 2, \dots N-1 \quad \textbf{(6)}$$

where after $N-1$ refraction events the beam has completed a full "circle".

A limiting factor may be the reflectivity of each block; whenever light passes through an interface, some of the beam will be reflected and some will be transmitted, therefore it is necessary to calculate the transmission coefficient at each interface in order to find the intensity lost due to reflection (the transmission coefficient is proportional to the intensity). Normally, as light travels through a medium it becomes further attenuated; in this case it is assumed that the blocks are thin enough that any intensity lost from this becomes negligible, and that the only energy lost is from the initial contact with each interface. Assuming the beam is plane polarised, the transmission coefficient, T, is equal to,

$$\begin{aligned} T &= 1 - R \\ &= 1 - \left|\frac{n_1\cos\theta_r - n_2\cos\theta_i}{n_1\cos\theta_r + n_2\cos\theta_i}\right|^2 \\ &= 1 - \left|\frac{\cos\theta_r - \frac{n_2}{n_1}\cos\theta_i}{\cos\theta_r + \frac{n_2}{n_1}\cos\theta_i}\right|^2, \end{aligned} \quad \textbf{(7)}$$

where R is the reflection coefficient [2]. The final expression is achieved by removing a factor of $\left(\frac{n_1}{n_1}\right)^2$ from the reflection term, allowing the incorporation of the constant from Eq.5. It can then be seen that the transmission coefficient for each new interface is constant, therefore this can also be raised to the power m to find the fraction of the original beam that has been transmitted after m refraction events.

By example, it can be seen that a very fine-tuned system would be required to produce a full quasi-circular path. In these examples, the beam will attempt to emulate a regular polygon of N = 100 sides (α = 176.4° via Eq.1). Using a θ_i value of 89°, θ_r = 85.4° from Eq.2 and C = 1.003 using Eq.5. From Eq.6 (where $m = 99$) the refractive index of the final block is found to be 1.36, which is similar to water [3]. However, each refraction event would transmit only 58.8% of the beam (via Eq.7), meaning that the beam rapidly attenuates. Conversely, if a θ_i value of 45° is used, 99.8% of the beam is transmitted through to the final block, but this would require a final refractive index of 756.5, which is obviously unphysical. Further analysis could potentially find an optimum polygon size and incident angle that would allow the system to work.

Conclusions

Aside from the discussed issues, the practice of creating a series of blocks with such fine-tuned refractive indices would prove difficult, and in reality the blocks could not be attached in such a way that dispersion at each interface could be eliminated.

A much simpler and more obvious way to achieve the same effect would be to use a series of mirrors, where θ_i could simply be chosen as half of the desired shape's interior angle, with each subsequent mirror being adjusted so that the reflected beam is incident to it at θ_i.

References

[1]http://www.regentsprep.org/regents/math/geometry/gg3/lpoly1.htm accessed on 31/10/14.

[2]http://en.wikipedia.org/wiki/Fresnel_equations accessed on 04/11/14.

[3]http://en.wikipedia.org/wiki/List_of_refractive_indices accessed on 17/11/14.