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A3_8 The Wings Of Pegasus

T. Bradley, G. McQuade, L. Garland and M. Walker

Department of Physics and Astronomy, University of Leicester. Leicester, LE1 7RH.

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Abstract

In this article a physical model is created to ascertain the size of the wings of the mythological flying horse Pegasus. Due to the nature of the physics involved, it was found that many assumptions had to be made that left some doubt for the quality of the final calculations. A minimum surface area of 7.9 m^2 was calculated.

The winged horse Pegasus [1] appears in Greek mythology and this article seeks to calculate the size of its wings. In order to do so, its wings are modelled as rectangular thin aerofoils (with negligible mass in comparison to it's overall body mass). The model seeks to calculate the lift generated with the wings flapping, though with lift only being produced via their properties as an aerofoil without considering how air is forced downwards by them. This can be done due to the fact that a bird (or other similar flying creature) will use their wings to generate thrust with the horizontal airflow due to their velocity providing lift. It is thought that for a minimum wing surface area Pegasus would run in order to take-off lending credibility to this assumption.

The equation for the force of lift (F_l) [2] produced by an aerofoil is given as,

$$F_l = \frac{1}{2} C_l A_S \rho v^2, \tag{1}$$

where C_l is the lift coefficient, A_S is the surface area of the aerofoil, ρ is the density of air and v is horizontal velocity. Aerofoils generate lift through their angle of attack (α). The angle of attack is the angle the aerofoil makes to the relative airflow (i.e if the wings are horizontally perfectly flat then the angle is 0). It is assumed due to the way in which an animals wings fold during an upstroke, there is a cancellation of this angle resulting in no effect (i.e the wings are bent in such a way that lift in any direction from the wings creates an overall net lift in any direction of 0). The effect of this angle is made clearer through through the relation [3],

$$C_l = 2\pi\alpha, \tag{2}$$

where the lift coefficient is proportional to the attack angle. In order to define the angle of attack, a dimensionless ratio known as the Strouhal number is used. This is given as [4],

$$\sigma = \frac{A_0 f}{v}, \tag{3}$$

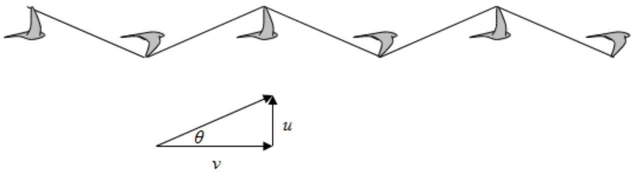

Fig. 1: [7] *assuming u is constant, a birds wings will oscillate upwards and downwards in a zig-zag pattern when compared with horizontal velocity with θ defining the angle.*

where σ is the Strouhal number, f is the frequency of wing flapping, A_0 is the amplitude of the wing flapping and v is the horizontal velocity.

It was found in a study that for efficient flying a bird, or in this case a winged horse, requires a Strouhal number between 0.2 and 0.4 [4]. Assuming that on each downstroke of wings there is a constant upwards velocity (u) achieved a Strouhal angle (θ) can be defined (Fig. 1). This angle defines the difference between upwards velocity generated during a downwards wingstroke and the horizontal velocity. To simplify the model it is assumed that Pegasus' wing oscillation speed is constant ($u = 2A_0 f$). Through substitution of this oscillation speed in Eq. 4, it can be seen that θ can then be related to the Strouhal number by further substitution using Eq. 3 (see Fig. 1).

$$\tan\theta = \frac{u}{v} = \frac{2A_0 f}{v} = 2\sigma. \tag{4}$$

Having defined the Strouhal angle the angle of attack for the wings can be defined. Assuming that Pegasus maintains its body position such that it's head faces forward at all times, it's wings must have 0 attack at the shoulders. It is also assumed that it's wingtips angle of attack will be equal to that of the Strouhal angle. A further assumption is that the attack angle increases along the wing linearly to the Strouhal angle giving the equation,

$$\alpha = 2\sigma \frac{r}{b}, \tag{5}$$

where b is the length of the wings and r is for any distance along the wings. Due to how small the Strouhal angle and angle of attack are, the trigonometric functions that are involved are neglected in order to simplify the model further.

Eq. 1 must be modified in order to use these factors. In order to do so the surface area term is adapted into an integral of dr between the limits of 0 and b (integrating over the length of the wing) with a factor c included defining the width of the wing.

$$F_l = 2\int_0^b \frac{1}{2} C_l c \rho v^2 \, \mathrm{d}r. \tag{6}$$

A factor of 2 appears so that the equation represents both wings individually. Substituting Eq. 2 and Eq. 5 and rearranging gives Eq. 7,

$$F_l = \frac{4\pi\sigma c \rho v^2}{b} \int_0^b r \, \mathrm{d}r. \tag{7}$$

Completing the integral then gives,

$$F_l = 2\pi A_S \rho \sigma v^2, \tag{8}$$

where A_S is substituted in for bc. This equation gives the lift force. Assuming that there is no effect during the upstroke of wings, and that the wings oscillate at a constant speed (both up and down) then this equation will only be valid for half the flight time. As of such, a time averaged lift force ($< F_l >$) is defined.

$$< F_l >= \pi A_S \rho \sigma v^2 \tag{9}$$

This time averaged value will be half that of Eq. 8 due to it only being in effect for half the flight time. Rearranging, the surface area can be defined in terms of the remaining parameters.

$$A_S = \frac{< F_l >}{\pi \rho \sigma v^2}. \tag{10}$$

The lift force can then be related to the weight of Pegasus ($< F_l >= Mg$, where M is the mass of Pegasus and g is acceleration due to gravity) which gives,

$$A_S = \frac{Mg}{\pi \rho \sigma v^2}. \tag{11}$$

The mass of a horse can vary between 400 kg and 850 kg [6] and it's running speed can vary between 30 mph and 50 mph [5]. It is assumed for this model that Pegasus' flying speed will be the same as its top running speed. This is due to the fact that in order to take-off Pegasus must be running resulting in a minimum wing surface area for a larger running speed. Considering Pegasus is a mythological horse with legendary abilities it is assumed that it can run at the larger of these speeds, $v = 50$ mph or in standard units $v = 22$ ms^{-1}. Also, air density can assumed to be that at sea level ($\rho = 1.225$ kgm^{-3}).

Using a mass of 600 kg (for a medium sized horse) it can be found that for a Strouhal number of 0.2, wing surface area A_S is 15.8 m^2 and for a number of 0.4, A_S is found to be 7.9 m^2. As of such a minimum surface area for each of Pegasus' wings is 7.9 m^2.

Conclusion

Though a surface area is calculated in this paper, due to the simplification of the physics involved it is highly debatable whether the value is accurate or reliable. The model developed relies on many simplifications which could be argued to be too many. Calculations were made using experimental values of the Strouhal number which could vary between a range that affects the surface area by a large amount due to the inverse relationship. Furthermore the validity of these values of the Strouhal number could come into question due to the size difference compared to other winged animals. Additionally the model created is flawed by the use of a large amount of assumptions which are not true though are used to simplify the physics involved. These include the assumption that there will be no effect during the upstroke of Pegasus' wings, the neglect of trigonometric functions while using angles and the estimates for mass and speed. However, it could be argued that the mass and speed assumptions can only ever be assumptions due to the fact that Pegasus does not actually exist.

References

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