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P5_2 Fluid Dynamics of a Near Supersonic Train

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Abstract
This paper investigates the effect on wind behaviour of allowing the trains of the London Underground to accelerate to near super-sonic speeds i.e. Mach 0.9. The paper calculates the magnitude of the piston effect of a deep underground train arriving at a platform on a human and finds this value to be 103.5N of constant force.

Introduction
Ever since its inception in 1863, the London Underground has been an invaluable method to traverse the capital. Over time, many deep tunnels have been constructed. Due to the enclosed nature of these tube tunnels, a phenomenon known as the piston effect occurs when a train moves through them. This effect is the source of the wind felt by passengers waiting on the platform as the train arrives and usually manifests as a gentle breeze. We investigate what would happen if the distance between stations was far greater than at present and thus, trains were to move at speeds close to the speed of sound. Disregarding the design and structural complications that would arise from this, we build a simple model for the magnitude of the piston wind.

Theory
The tunnel is modelled as a cylinder of radius r. It is also assumed that the train dimensions are such, that there is a vacuum seal between the train and tunnel walls. This allows us to ignore any complications which arise from air flow around the sides and top of the train. We can then model this situation as a piston moving through a cylinder. Upon reaching the exit of the tunnel, the train enters an enclosed platform area of radius R, which is approximately equal to three times r (this holds true for many deep underground stations)[1][2]. The air in the tunnel is taken to be an incompressible fluid and thus the following equation for mass conservation can be applied,

$$\rho v_1 A_1 = \rho v_2 A_2 \tag{1}$$

Where ρ is the density of the air, v_1 and A_1 are the velocity and area inside the tunnel and v_2 and A_2 are the velocity and area on the platform. This equation can be rearranged to find v_2, the velocity of the air at the platform.

$$v_2 = \frac{v_1 A_1}{A_2} \tag{2}$$

Although the air on the platform is moving at velocity v_2, in order to calculate the force felt by a person standing on the platform, we need to consider a force analogous to drag. The drag force, F_d experienced by a falling object is as follows,

$$F_d = \frac{1}{2} \rho v^2 C_d A \tag{3}$$

Where ρ is the density of the medium (in this case, air), v is the velocity of the object, A is the effective area of the object and C_d is the coefficient of drag (which is dependent on the shape of the object). We consider the case of a free falling human, in which case the drag force is simply mg, with m being the mass of the human, taken as 70kg and g being the acceleration due to gravity, taken as 9.81 ms^{-2}. Eq. (3) is then rearranged to find the drag coefficient for a human, taking v to be the terminal velocity of 56 ms^{-1}, ρ to be 1.225 kgm^{-3} (the density of air) and the effective A to be $0.5m^2$ (applicable in our case of a standing human facing the wind)[3]. The drag coefficient is calculated to be 0.715.

Eq. (2) can then be substituted into Eq. (3) to find the force experienced by a human standing on the platform. The final equation for the force is found to be,

$$F=\frac{\rho C_d A}{2}\left(\frac{v_1 r^2}{R^2}\right)^2 \tag{4}$$

Where all the variables are as defined previously. In this case, ρ is taken as 1.225kgm^{-3}, C_d as 0.715, A as $0.5m^2$, r as 1.25m[4], R as 3.75m (3r) and v_1 as 306.3ms^{-1} (Mach 0.9). The result is found to be 103.5N.

Discussion

A force of 103.5N would be equivalent to approximately 10kg of force being applied to a standing person. This is enough to unbalance a person, forcing them to take a step to steady themselves. The potential for harm comes from the environment of the platform, a piece of luggage could lead to someone tripping and injuring themselves and others around them.

Conclusion

For the reasons discussed above, we feel that London Underground trains cannot safely travel at such high speeds without additional measures to protect from the piston wind on platforms. Some platforms currently have screens built into them to prevent suicides. We believe these screens could be a measure to look into for protection from the piston wind.

References

[1]http://commons.wikimedia.org/wiki/File:Piccadilly_Circus_tube_stn_Piccadilly_eastbound_look_west.JPG (20/10/2014)
[2]http://en.wikipedia.org/wiki/London_Underground#mediaviewer/File:Lancaster_Gate_tube.jpg (20/10/2014)
[3]J. McIlveen, 2002, Weather Vol. 57
[4]http://www.tubeprune.com/rollingstock-outline.html (20/10/2014)