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A4_7 Supermassive (Interstellar) Black Hole

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Abstract
This paper considers the plausibility of an event from the 2014 film *Interstellar*. It states that an hour on a planet orbiting a supermassive back hole is equivalent to 7 years passing on Earth. Calculations made state that the planet is inside the minimum orbit for a planet around a black hole and would thus fall with a ballistic trajectory into the centre. However, a static system was assumed, so with an optically spinning black hole the planet may be in a stable orbit. A spacecraft orbiting at a distance unaffected by time dilation is also considered and found to be largely implausible.

Introduction

Interstellar follows astronauts travelling to potentially habitable planets in a new system brought conveniently close to Earth via a wormhole located near Saturn. One such planet known as *Miller's Planet* orbits a supermassive black hole and as such is affected by gravitational time dilation. When the astronauts spend an hour on the surface of the planet, 7 years elapse not only on Earth but on their mothership which is staying outside the effect of the gravitational potential well.

Other important factors when determining the plausibility of the system are the location of the minimum stable orbit and the Schwarzschild radius, also known as the event horizon, within which nothing, not even light, can escape. Figure 1 illustrates the system. We are not considering the repercussions of the planet orbiting at relativistic velocities, only the time dilation due to the gravity of the black hole.

When does 1 hour equal 7 years?

The equation for gravitational time dilation for a circular orbit is given by [1];

$$T = \frac{T_o}{\sqrt{1 - \frac{3}{2}\frac{r_s}{r}}}, \qquad (1)$$

where T_o is the observed time, T is the time observed from far away (i.e. Earth), r_s is the Schwarzschild radius and r is the orbital radius. Using the definition of the Schwarzschild radius [2],

$$r_s = \frac{2GM}{c^2}, \qquad (2)$$

we can rearrange eqn.1 to find the orbital radius for a given time dilation,

$$r = \frac{3}{2}\frac{2GM}{c^2(1 - (\frac{T_o}{T})^2)}. \qquad (3)$$

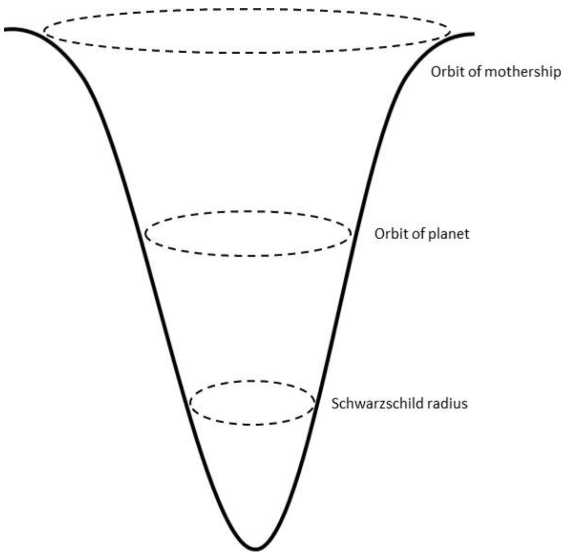

Fig. 1: The structure of the gravity well around the black hole. Distances not to scale

G=6.67×10^{-11}m^{3}kg^{-1}s^{-2} is the gravitational constant, c=3×10^8ms^{-1} is the speed of light and M is the mass of the black hole stated by the film producers to be 100 million $M_\odot$ where the mass of the sun ($M_\odot$) is 1.99×10^{30}kg [3]. For T_o=1 hour and T=24×365×7 hours, the orbital distance is calculated to be 4.42×10^{11}m (2.95Au). Using eqn.2 the Schwarzschild radius for this supermassive black hole is found to be 2.95×10^{11}m (1.96Au). For a structure 100 million times the mass of the sun, 1Au from the planet to the event horizon is a tiny distance.

What counts as a safe distance away?

The mothership stationed outside the effect of gravitational time dilation is next considered. A small amount of time dilation is taken to be the amount experienced by astronauts on the ISS. This can be found by putting the orbital distance of the ISS (∼400km) [4] and the Earth's mass (5.97×10^{24}kg) into eqn.1 (including r_s from eqn.2). This gives a 17ns per second time difference with Earth. Using this to set T=1+(1.7×10^{-9})s and T_o=1s eqn.3 can be used to find an extremely safe orbital radius from the black hole. This is found to be 1.3×10^{19}m which is 86.8 million Au from the centre! For a more achievable safe distance the time is set to run twice as fast (T=2T_o) giving an achievable safe distance of 5.90×10^{11}m (3.93Au).

The time it would take to travel from the planet to these safe distances can be found from hints in the film. It is stated that it takes the astronauts 2 years to travel from Earth to Saturn. We can use this to find a rough estimate for the speeds at which they can travel. For the highest possible speed and therefore the most optimistic travel times it is assumed that Saturn is at its furthest distance from Earth (1.7 billion km) [5][Note: At this distance Saturn would be directly behind the Sun so the path could not be direct]. Converting 2 years into 6.31×10^{7}s the average speed is approximately 27.0kms^{-1}. The time to travel from the planet at an orbital distance of 2.95Au to the extremely safe distance (86.8 million Au) is therefore 15 million years! For the achievable safe distance (3.93Au) it would take 63 days. This makes the ability to quickly visit the surface with minimal time change on return to the ship apparently impossible within the technology of the film.

Minimum stable orbit

Fabian et al. (2000) discusses the limits for the minimum stable orbit around black holes. It is referred to as the radius of marginal stability (r_{ms}), within which matter would follow a ballistic trajectory into the centre of the black hole. For a static black hole r_{ms}=6r_s. For the black hole considered here r_{ms}=1.77×10^{12}m (11.8Au) using r_s from eqn.2. This is much further out than *Miller's Planet* at 2.95Au, therefore the planet could not exist in its orbit for a static black hole. Fabian et al. also discuss how a spinning black hole effects the minimum orbit. For prograde orbits and an optimally spinning black hole r_{ms}=1.235r_s [7]. This gives a minimum orbit of 3.64×10^{11}m (2.43Au). *Miller's Planet* would therefore be in an acceptable orbit for a maximally spinning black hole.

Conclusion

Interstellar is very accurate for a Hollywood film, going so far as to consult with physicist *Kip Thorne* [3] to improve the realism of the film. However, no film is flawless, and what has been shown here is that some elements still require a suspension of disbelief.

Although a rotating black hole may legitimize the portrayal of *Miller's Planet*, the affects of time dilation while travelling to and from the planet do not hold up under close scrutiny. This is due to the 63 day travel time to reach a distance with relatively small time dilation. The elapsed time depicted in the film on the astronauts' return to the mothership should therefore be significantly greater. This said, it is still felt that the film should be applauded for showing a respect to science not often seen in cinemas.

References

[1] `http://en.wikipedia.org/wiki/Gravitational_time_dilation` Accessed on 12/11/2014

[2] `http://hyperphysics.phy-astr.gsu.edu/hbase/astro/blkhol.html` Accessed on 12/11/2014

[3] `http://www.space.com/27692-science-of-interstellar-infographic.html` Accessed on 12/11/2014.

[4] `http://www.esa.int/Our_Activities/Human_Spaceflight/International_Space_Station/ISS_International_Space_Station` Accessed on 12/11/2014

[5] `http://www.space.com/18477-how-far-away-is-saturn.html` Accessed on 12/11/2014

[6] `http://ned.ipac.caltech.edu/level5/Fabian4/Fab4.html` Accessed on 12/11/2014

[7] A.C. Fabian, K. Iwasawa, C.S. Reynolds, A.J. Young, *Broad iron lines in Active Galactic Nuclei* PASP, vol 112 (2000)