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P5_4 Running in the Rain

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Abstract

This paper investigates the degree of wetness that a person would experience whilst travelling on foot through the rain in various weather conditions. It finds that moving as fast as possible when travelling during conditions with no wind, a headwind and a light tailwind is best to minimise how wet one gets. It also finds that in the case of a strong tailwind, it is best to match the speed of the wind to minimise this.

Introduction

There exists a common dispute as to how wet a person gets whilst travelling at different speeds through the rain and which approach is best. The argument is that a person moving at a higher speed will pass through more volume of rainfall but stay in the rain for a shorter time.

Here, the "wetness" of an object will be determined by the number of rain drops that pass through the surface area of the object. This value shall be determined by considering flux of rain droplets.

Theory

Flux through a surface is defined as,

$$\vec{F} = \frac{\partial N}{\partial t \partial A}\hat{\imath} \qquad (1)$$

In this case *N* is the number of rain drops, t is time and *A* is area with $\hat{\imath}$ being a unit normal. Rearranging for *N* from (1) we obtain,

$$N = \iint \vec{F} \cdot \hat{\imath}\, dAdt \qquad (2)$$

Here, we consider flux through two areas, from the front ($\vec{F}_h$) and from above ($\vec{F}_v$) with the subscripts standing for horizontal and vertical respectively. It can be clearly seen that the total flux will be the sum of these values.

We can calculate these fluxes using the following calculations.

$$\vec{F}_h = nv_p \qquad (3.1)$$

$$\vec{F}_v = nv_R \qquad (3.2)$$

Where v_p is the velocity of the person, v_R Is the velocity of the rain and n is the number density of rain drops.

However, this does not account for the angle of the falling rain. If the rain is falling at ϑ to the vertical, then $\vec{F}_v$ can be expressed as,

$$\vec{F}_v = nv_R\, cos(\theta) \qquad (4)$$

In this equation, $v_R\, cos(\theta)$ is the vertical velocity component of the rain.

It is apparent that if an object is moving at the same velocity as the horizontal component of the rain then there will be no horizontal flux.

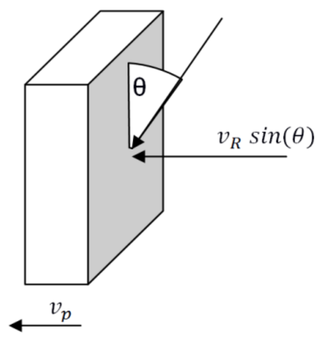

Figure 1 shows the condition where $v_R\ sin(\theta)$ and v_p are in the same direction.

Hence the expression for $\vec{F}_h$ is

$$\vec{F}_h = n\left|v_p - v_R\, sin(\theta)\right| \qquad (5)$$

The modulus is present because regardless of either v_p or $v_R sin(\theta)$ being greater, the flux is positive.

If v_p and n are taken as constant then, considering v_R is also constant, the integral in equation (2) can be evaluated giving,

$$N = nA_v v_R\, cos(\theta)t + nA_h\left|v_p - v_R\, sin(\theta)\right|t \qquad (6)$$

The time spent in the rain is governed by the velocity of the person and how far away they are from shelter. A substitution can be made for time by writing it as $\frac{x}{v_p}$, where *x* is distance. The expression can be simplified further if we consider that the vertical velocity is the terminal velocity of a rain drop and the horizontal velocity of a rain drop is approximately equal to component of the wind speed in the direction of motion. This allows for the substitutions $v_R\,cos(\theta) = v_T$ and $v_R\,sin(\theta) = v_W$, where v_T is the terminal velocity and v_W is the wind velocity in the direction of motion. These substitutions give the number of rain drops encountered as

$$N = nA_v x\frac{v_T}{v_p} + nA_h x\left|1 - \frac{v_W}{v_p}\right| \quad (8)$$

This equation can be considered under three different conditions, given by $\frac{v_W}{v_p}$ being greater than, less than and equal to zero.

Discussion

The first scenario described by equation (8) is that of a strong head wind. In this case, v_W will be in the opposite direction to v_p and thus negative. Therefore, the modulus term $1-\frac{v_W}{v_p}$ will always be greater than 1 and will have an asymptote at 1 when $v_p \rightarrow \infty$. This can be seen in *Figure 2*.

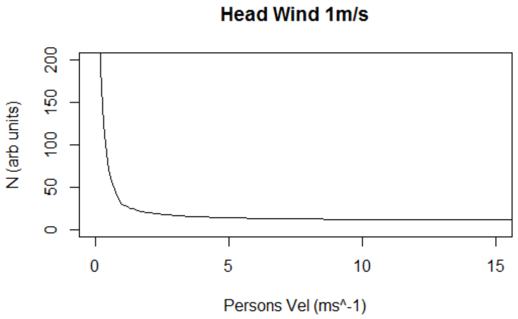

Figure 2 shows that the rain drops encountered will asymptote to a finite value. This graph gives N in arbitrary units since an accurate calculation would be no more informative.

From this, we can see that with a headwind, the faster one runs, the fewer rain drops they will encounter.

The second scenario is that of no wind, which gives the condition $v_W = 0$. Since n, A_h and x are all constants, the modulus term becomes constant. Again, the least amount of rain is encountered by moving as fast as possible. This is shown in *Figure 3*.

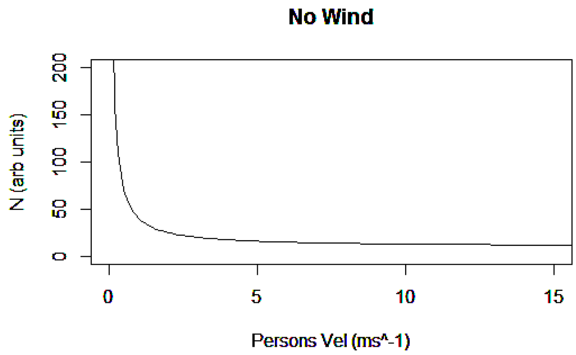

Figure 3 shows the same result as Figure 2.

The final scenario is that of a tailwind. In this case, v_W is positive. Under this condition, it is found that the modulus term of equation (8) will reach a minimum of zero when $v_W = v_p$. However, this does not necessarily lead to a minimum. If v_W is negligible then we find that there is no minimum at finite velocity.

A minimum does exist if the coefficient for the modulated $1/v_p$ is greater than the coefficient for the unmodulated $1/v_p$, or when $A_h v_W > A_v v_T$.

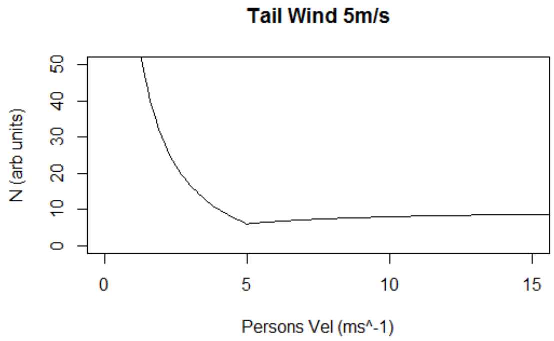

Figure 4 shows a minimum when co-moving with the wind, as expected.

Conclusion

In conclusion, the answer to this debate is entirely dependent on the wind conditions one is travelling through. In the majority of situations, moving as fast as possible is more beneficial but of course, this is not found to be the case with a strong tailwind.

References

No references used.