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P5_3 Liquid Lens

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Abstract

This paper investigates the possible manipulation of the capillary effect to manufacture small disposable lenses from a fluid. It concludes that, through the process of refraction, light can be made to focus or disperse. The factors that decide the behaviour of the light as it crosses the interface are identified as the ratio of refractive indices and the differences in surface tension.

Introduction

Observations of the capillary action were first documented by Leonardo da Vinci [1]. The observation made by da Vinci was how, when a glass pipe was lowered into water, the water within the pipe would rise above the outside water's surface and appear to climb a short height up the glass's sides. This phenomenon is known as a meniscus and is common to all fluids held in a container.

If the container is a narrow tube, the rises at the interface of the fluid and the container meet in the centre of the tube and form a lens-shaped surface. Whether or not this will occur can be guesses by calculating the bond number B_o[2],

$$B_o = \frac{\rho g a^2}{\sigma_{lg}} \quad (1)$$

Where ρ is the density difference between the two fluids, g is acceleration due to gravity, a is the radius of the capillary tube and σ_{lg} represents the surface tension between the liquid and gas. The bond number compares the forces of surface tension to the forces acting on the fluids body like the fluids weight; a low bond number (<1) indicates the possible formation of a meniscus. The surface formed can be convex or concave in nature, depending on the surface tensions between the liquid, container and the fluid above the liquid.

Theory

One of the implications of equation (1) is that almost all fluids will form a meniscus if the capillary tube is sufficiently narrow. The interaction between the incident light and the fluid interface is given through Snell's law of refraction.

$$\frac{\sin\varphi_1}{\sin\varphi_2} = \frac{n_2}{n_1} \quad (2)$$

Where φ refers to the angle between the light plane wave and the normal to the interface surface and n refers to the refractive indices of the substances on either side of the interface. The subscripts 1 and 2 denote different sides of the interface between the substances; for this paper the subscript 1 will represent the side of the interface where the light is originating from.

The ratio of refractive indices is one of two factors that will dictate whether the meniscus will cause the light incident on its surface to focus or disperse. The other factor is the contact angle between the fluid and the capillary tube, γ_c, which is given by equation (3)[3].

$$\cos\gamma_c = \frac{\sigma_{wg} - \sigma_{wl}}{\sigma_{lg}} \quad (3)$$

In this equation the σ terms denote the surface tension between the two substances specified by their subscripts; w for wall, g for gas and l for liquid. Equation (3) dictates if the meniscus of the fluid is convex or concave in nature. If the numerator of (3) is positive then the contact angle, meaning the meniscus is concave as is the case with a water-glass-air system. If the numerator is negative then the meniscus is convex, this is the case with a mercury-glass-air

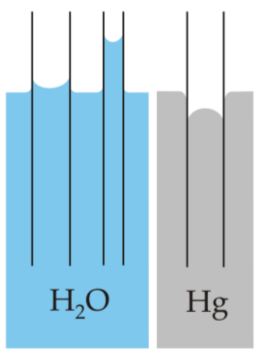

Figure 1: Illustration of the menisci of water and mercury. [4]

system, both cases are shown in figure (1).
The two factors that decide if the light is dispersed or focused have now been identified and their effects understood quantitatively. Table (1) summarises the result directing a plane wave of light on a meniscus.

	$\frac{n_2}{n_1} > 1$	$\frac{n_2}{n_1} < 1$
$(\sigma_{wg} - \sigma_{wl}) > 0$	Focus	Dispersion
$(\sigma_{wg} - \sigma_{wl}) < 0$	Dispersion	Focus

Table 1: Summary of a meniscus's effects on an incident ray of light.

Until now the concern of this paper has been the effects of a meniscus on a plane wave of light across its entire front, now however, the effects on a single ray that is vertically incident on the interface will be considered. For this to occur, a method of singling out a source of light from anywhere between the centre of the tube and its wall at radius a, and finding its contact angle with the interface is required. This problem is shown geometrically in figure (2).

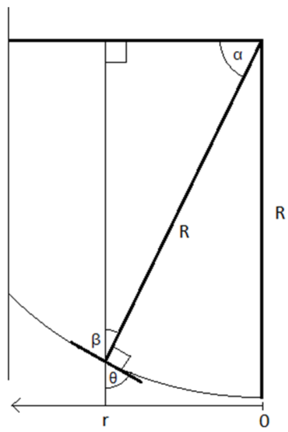

Figure 2: A diagram of the contact angle from a variable distance across the capillary tube radius.

The quantity for which a value is required is *θ,* the angle between the instantaneous gradient of the meniscus's curve and the vertical i.e. the direction the approach of the light ray. It can be seen from figure (2) that the angle between curve normal and vertical, *β*, is part of a right angle triangle. This means that the angles *α* and *θ* have the same value. Angle *α* can be stated using the cosine function, giving the formula $\cos\theta = \frac{r}{R}$ to describe *θ* as a function of *r*.

Discussion

Now that an expression for the incident angle has been derived, the next step is to use this information to generate a formula that provides the direction of travel of the transmitter light ray. Since the incident angle of the light ray from equation 2, φ_1, and *θ* make up a right angle, φ_1 is equal to *90-θ*. Through further trigonometric manipulation it is revealed that $\sin\varphi_1 = \cos\theta$.
Using the formula for *cosθ* derived in the previous section and the identity shown in the above paragraph, a new form of equation (2) can be shown.

$$\sin\varphi_2 = \frac{rn_1}{Rn_2} \qquad (4)$$

Equation (4) provides a way of predicting the path a ray of light that is incident on the interface of a meniscus from the vertical direction will take after it has been transmitted.

Conclusion

While the theory behind the use of the capillary action to focus or disperse light, almost no applications for which such a process would be valid are known to exist. Since meniscus only form over small radii, the possible power output of this device would be very small. Other than liquid water and air, it would be challenging (and probably hazardous)to acquire and use other liquids and gasses for the purpose of light focusing or dispersal.

References

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[4]http://en.wikipedia.org/wiki/Capillary_action accessed on 19/11/14