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P1_5 Combating Global Warming With Orbital Transfers

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Abstract

In this paper, the orbital radius required to cool the Earth by 2K is determined using the Stefan-Boltzmann Law. The delta-V required to achieve this new orbital radius using a Hohmann Transfer manoeuvre is then determined and the mass of fuel required for such a manoeuvre is determined using Tsiolkovsky's rocket equation. The total mass of fuel required is found to be 0.04374 Earth masses.

Introduction

This paper investigates the mass of fuel required to place the Earth onto a circular orbit with a radius sufficient to cool the Earth's effective temperature by 2 K through the use of an orbital manoeuvre known as a Hohmann transfer.

Required Orbital Radius

The Stefan-Boltzmann Law relates the power radiated by a black body to its effective temperature and is given by

$$L = A\sigma T^4,$$

where *L* is the power, *A* is the surface area of the black body and *T* is the body's effective temperature[1]. Assuming the body is radiating isotropically, the radiative flux per unit area at a distance of *r* from the radiating black body will be given by

$$F = \frac{L}{4\pi r^2}.$$

If the Sun and Earth are considered to be black bodies in thermal equilibrium, then the energy absorbed by the Earth is given by

$$E = \pi R_{\oplus}^2 \frac{R_{\odot}^2 \sigma T_{\odot}^4}{r^2},$$

where $R_{\oplus}$ is the radius of the Earth, $R_{\odot}$ is the radius of the Sun, $T_{\odot}$ is the Sun's effective temperature and *r* can be taken to be the Earth's orbital radius. For the Earth to be in thermal equilibrium the amount of energy it emits must equal the amount it absorbs, therefore

$$4\pi R_{\oplus}^2 \sigma T_{\oplus}^4 = \pi R_{\oplus}^2 \frac{R_{\odot}^2 \sigma T_{\odot}^4}{r^2},$$

and terms can be cancelled and the equation rearranged to find the Earth's temperature

$$T_{\oplus} = T_{\odot}\sqrt{\frac{R_{\odot}}{2r}} \quad (1).$$

For the purposes of this paper, it is assumed that global warming has a purely additive effect on the Earth's temperature, and that the Earth perfectly absorbs all solar radiation incident on it (its albedo is 0). The Earth's current effective temperature is to be found and equation (1) is to be rearranged to find a value of *r* such that this effective temperature is reduced by 2 K.

$T_{\odot}$ is taken to be 5780 K, *r* is taken to be 1 AU, or 1.49598×10^{11} m, and $R_{\odot}$ is taken to be 6.96×10^{8} m. Using these values in equation (1) we find the Earth's effective temperature to be 279 K. Rearranging equation (1) to be in terms of Earth-Sun separation provides the following

$$r = \frac{R_{\odot}}{2}\left(\frac{T_{\odot}}{T_{\oplus}}\right)^2 \quad (2).$$

Taking $R_\odot$ and $T_\odot$ to be the previous values and $T_\oplus$ as 277K we obtain a value for Earth-Sun separation of 1.51522x10^{11} m, which can be considered to be the new Earth orbital radius.

Orbital Transfer

In order to reach this new orbital radius, the Earth will be placed onto a Hohmann Transfer orbit. Two impulses (assumed to be instantaneous) are used: one to place the Earth onto an elliptical transfer orbit and another is executed at the transfer orbit apoapsis to circularise at the new orbital radius. The two changes in velocity *Δv* required to place the Earth onto these orbits and the total *Δv* required can be determined using the following equations

$$\Delta v_1 = \sqrt{\frac{GM_\odot}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right) \quad (3),$$

$$\Delta v_2 = \sqrt{\frac{GM_\odot}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right) \quad (4),$$

$$\Delta v_{tot} = \Delta v_1 + \Delta v_2 \quad (5),$$

where r_1 is the orbital radius of the Earth found using equation (1), r_2 is the orbital radius of the Earth found using equation (2) and $M_\odot$ is the mass of the Sun, assumed to be 1.98855x10^{30} kg[2]. Substituting values previously calculated into equations (3), (4) and (5) yields a required Δv_{tot} of 189 ms^{-1}

Mass of Fuel Required

It is assumed that the impulses to transfer the Earth onto these orbits are provided by a liquid bipropellant rocket engine. The rocket engine, the fuel to power it and the Earth are considered to be a single “spacecraft”. Tsiolkovsky’s rocket equation relates the maximum Δv that can be accomplished by a spacecraft powering a rocket engine with a given mass of fuel and is expressed as

$$\Delta v = I_{sp} g_o \ln\frac{m_o}{m_1},$$

where I_{sp} is the rocket’s specific impulse, g_0 is the acceleration of free fall at the Earth’s surface and m_0 and m_1 are the rocket’s initial and final mass respectively [3]. This can be rearranged to find the ratio between these two masses

$$\frac{m_o}{m_1} = e^{\left(\frac{\Delta v}{I_{sp} g_0}\right)}.$$

If the final mass is the mass of the Earth (5.97219x10^{27} kg) and the rocket’s specific impulse is taken to be 450s (a typical value for a bipropellant liquid fuel rocket [4]), the ratio between the masses is found to be 1.04374. This means the mass of fuel required to power the rocket is 0.04374 Earth masses, or 2.6124x10^{26} kg. To put this in perspective, if the two propellants used are liquid hydrogen and liquid oxygen, electrolysing the total mass of water on Earth (1.4x10^{21} kg [5]) would only yield 0.000536% of the fuel required to perform the required burn.

References

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