

## A5\_6 Pyramid of Geezers

H. J. Allison, J. I. Penney, R. P. Leyser, G. C. Lipscombe

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

November 6, 2014

### Abstract

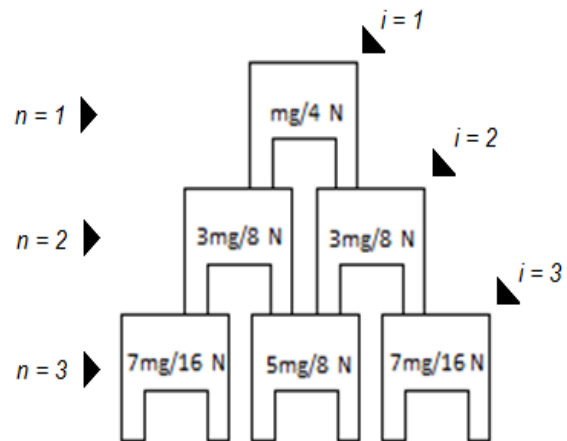
An investigation is conducted into the maximum size of the popular challenge, the ‘human pyramid’. Under the assumption that the formation consists of people with perfect balance, the limitations upon the size therefore arise from the strength of the individuals. It is found that for a pyramid consisting of males of uniform mass, the maximum height is four tiers, potentially five if the individuals were marginally stronger than average. For a pyramid of men, women, and children, assembled with the highest mass on the base, the maximum achievable height was six tiers.

### Introduction and Theory

A well-known and popular challenge is the ‘human pyramid’, a gymnastics formation whereby individuals kneel on top of one another in rows of gradually decreasing length. A simple diagram of the formation is shown by figure 1. This party game is a test of both balance and strength, where typically the strongest, heaviest people form the bottom few rows. An investigation is performed to determine the theoretical maximum size of a ‘human pyramid’ assuming that the individuals involved have perfect balance. Under this assumption the achievable size is limited by the weight that the bottom tier can support.

Initially, it is assumed that the pyramid is constructed of a number of identical males, each of mass 83.6kg (the average mass of an adult male [1]). This result is then compared to the maximum size obtained from the logical order of heaviest people on the bottom and lightest on the top. As an extreme example, children were assumed as the lightest people (32.2 kg), positioned in the top layers; women (70.2 kg), of moderate weight, in the middle; and men located on the bottom levels. Where the number of tiers is divisible by three, the number of levels of children, women, and men are equal.

The legs and arms of each person within the pyramid are collectively required to support half of the weight supplied by the two people above, in addition to the person’s own weight.



**Figure 1** – A diagram of the arrangement of a human pyramid. Each ‘leg’ shown represents an arm and a leg. The force on each limb is displayed at each position in the pyramid, assuming a uniform mass of individuals. Lines of constant  $n$  and  $i$  are displayed. Lines of constant  $i$  are diagonal and each  $n$  value corresponds to a tier in the pyramid.

It is assumed that the total weight on an individual is equally distributed about all four of the person’s limbs. The weight,  $F_g$ , of each person is given by:

$$F_g = mg \quad (1)$$

where  $m$  is the mass of the individual and  $g$  is the acceleration due to gravity ( $9.81 \text{ ms}^{-2}$ ).

For notation purposes, the pyramid is divided into  $n$  rows, with each position in the row denoted by an integer  $i$ , as shown in figure 1. It should be noted that lines of constant  $i$  run diagonally down the pyramid; arrows in figure 1 show the direction of these diagonal lines. The

$i^{\text{th}}$  member of the  $n^{\text{th}}$  row is required to support a force of  $F_{limb}(n, i)$  per limb. This is given by:

$$F_{limb}(n, i) = \frac{mg}{4} + \frac{2F_{limb}(n-1, i)}{4} + \frac{2F_{limb}(n-1, i-1)}{4} \quad (2)$$

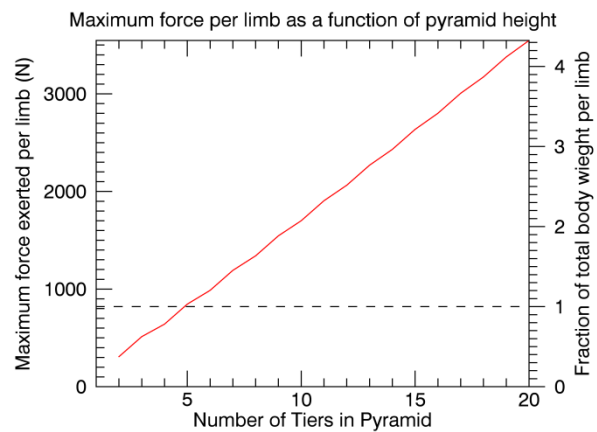
In this equation,  $F_{limb}(n-1, i)$  is the force on each limb of the individual located in the row above (row  $n-1$ ) but at the same  $i$  position. Likewise,  $F_{limb}(n-1, i-1)$  denotes the force per limb of the individual in the row above at the  $i-1$  position. The factor of 2 is included in front of each of these terms as the person above will have two limbs on the individual below. It may be the case that either  $F_{limb}(n-1, i-1)$  or  $F_{limb}(n-1, i)$  is zero as the  $i^{\text{th}}$  person of row  $n$  may be located in the first or last position respectively. At the top of the pyramid ( $n = 1$ ) both these terms will be zero. Using equation (2) iteratively, the force on *each* limb has been included in figure 1, for a pyramid consisting of people of identical mass.

**Results and conclusion**

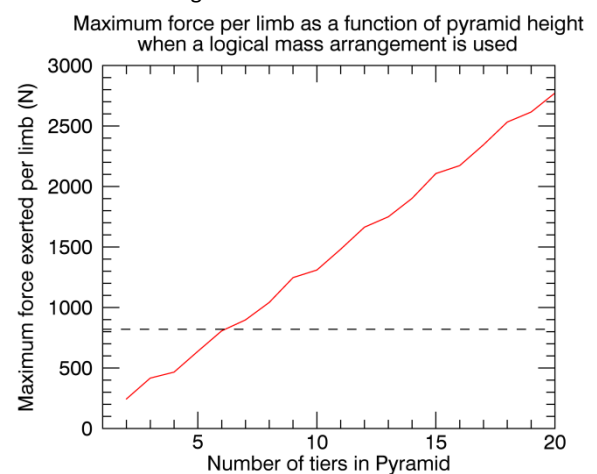
The person, or people, in the centre of the bottom row have limbs subjected to the greatest force; it is in this location that the pyramid will falter. In light of this, from this point onwards, only the force on each limb felt at this position is considered. Figure 2 is a graph of how, at this location, the force per limb changes with increasing pyramid height. It is suggested that the average male is capable of performing a bench press of his own weight [2]. Considering this, it is assumed that the male arm is capable of supporting (not lifting) his own body weight. From equation (1), the limiting weight that the arm can support is then ~820 N. Legs are capable of supporting more weight (one can easily stand on one leg holding an item) so arm strength is considered as the limiting factor.

Examining figure 2, it can be seen that the maximum height of a pyramid of identical males is four tiers. If the individual (individuals) located in the centre of the bottom row is stronger than average it is possible that a five tier pyramid could be achieved. For a five tier pyramid the force per limb at this location is only marginally more than the maximum value.

In comparison, figure 3 shows that a pyramid constructed of men, women, and



**Figure 2** – Graph of how maximum force per limb changes with increasing pyramid height for a pyramid constructed of identical men. A dashed line is included to demonstrate the assumed strength of an arm.



**Figure 3** – Graph showing the maximum force per limb for pyramids of varying height with weight distributed with the lightest individuals on the top and the heaviest at the bottom. A dashed line is included to demonstrate the assumed weight which a male arm can support.

children may reach a greater height before faltering. In this scenario, a six tier pyramid is possible.

A trend displayed by both figure 2 and 3 is a greater force increase when changing from an even number of tiers to an odd number of tiers. This is because the maximum force per limb is experienced by two people in an even tier pyramid. This suggests that if the people in the bottom layer have arms capable of supporting more than their own body weight the pyramid is more likely to reach another tier if the previous maximum number of layers was odd.

**References**

[1] <http://www.bbc.co.uk/news/uk-11534042>, 2010, accessed on 29/10/14  
 [2] <http://hypertextbook.com/facts/2003/AlexSchlessingerman.shtml>, accessed on 05/11/2014