Journal of Physics Special Topics

A5_10 An Interstellar Mess Up?

J.I. Penney, G.C. Lipscombe, R.P. Leyser, H.J. Allison

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

November 19, 2014

Abstract

The paper investigates the physical implications behind the recent film Interstellar, surrounding the tidal forces acting on the planets orbiting the Supermassive Black Hole. The paper determines the minimum radii for each system to approach the structure, and finds that Earth-like planets could exist within the system. It is also found that the crew would be able to traverse the event horizon without damage to the spacecraft.

Introduction

The film "Interstellar" depicts several planets in a close orbit about a Supermassive Black Hole (SMBH). Many critics have argued that these planets could not exist due the massive gravitational tidal forces from the structure. This paper investigates the accuracy of such claims by looking at the forces from the SMBH compared to the gravitational field of the planet, which is assumed to be Earth-like in mass, and determine at what radius it could orbit before becoming tidally disrupted. We assume that the SMBH in question is in a quiescent state [1] such that irradiation of the planet to beyond the point of habitability is not considered. Different average densities for the planet have been considered.

Additionally, the paper investigates the mass at which the tidal disruption from the SMBH would overcome the forces maintaining the ship's integrity, where the ship, "*Endurance*", is assumed to be of similar proportions to a Saturn V rocket [2]. This can then be used to determine the closest distance of approach for the crew and the ship.

Theory

To determine the force of disruption from the SMBH on a body, the difference in forces between the nearest point and the furthest point of the object must be taken into account, as determined in below:

$$\Delta F = \frac{GM_{BH}m_p}{(r+R)^2} - \frac{GM_{BH}m_p}{r^2}, \qquad (1)$$

where M_{BH} is the mass of the SMBH, r is the radius between the body and the SMBH, R is the radius of the body, and m_p is the mass of some small increment of the body. Rearranging Equation (1):

$$\Delta F = \frac{2GM_{BH}m_p(R^2-2rR)}{r^2R^2+2Rr^3+r^4} \qquad (2)$$

Assuming that $R << r$ and neglecting lower order terms of r in the denominator:

$$\Delta F = \frac{2GM_{BH}m_pR}{r^3}. \qquad (3)$$

To tidally disrupt a large mass held together by its own gravity, such as a planet or star, the tidal force in Equation (3) must be greater than the gravitational forces holding the object together:

$$\frac{2GM_{BH}m_pR}{r^3} > \frac{Gm_{obj}m_p}{R^2}, \qquad (4)$$

where m_{obj} is the mass of the object in question. Rearranging Equation (4) for M_{BH} gives

$$M_{BH} = \frac{r^3 m_{obj}}{2R^3} \;. \qquad (5)$$

The radius, R of the object and its mass are related by

$$m_{obj} = \frac{4}{3}\pi R^3 \rho_{obj}\,, \qquad (6)$$

where ρ_{obj} is the density of the object in orbit about the central mass [3]. The minimum radius for the orbit of the planet around the SMBH can be given by the Schwarzschild radius, shown below:

$$r_{sch} = \frac{2GM_{BH}}{c^2} \;, \qquad (7)$$

where r_{sch} is the Schwarzschild radius. Substituting for r in Equation (5), and using the mass-radius relation for a planet:

$$M_{BH} = \sqrt{\frac{3c^6}{16\pi\rho_{obj}G^3}}\,. \qquad (8)$$

From this, it is possible to find the mass of the SMBH for the objects in question and determine whether Earth-sized objects could have orbited the SMBH seen in the film [4].

The ultimate tensile strength is used to determine the force needed to rupture the hull from the tidal disruption of the ship. This is defined by:

$$UTS = \frac{F}{A}, \quad (8)$$

where *UTS* is the ultimate tensile strength of the hull, assumed to be roughly carbon fibre [5] and *A* is the cross sectional area of the craft. The force is equated to Equation (3) and rearranged to obtain the maximum radius of approach:

$$r = \sqrt[3]{\frac{2GM_{BH}m_{ship}\Delta r}{(UTS)(A)}} \quad , \quad (9)$$

where Δr is the height of the spacecraft and m_{ship} is the mass of the ship.

Results

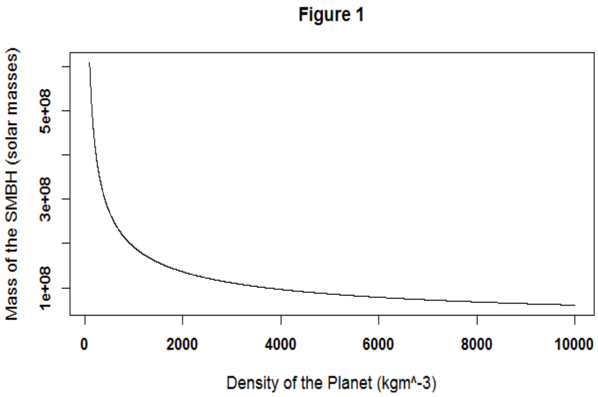

Figure 1. Plots the Mass of the SMBH in solar masses as a function of the density of the planet. It should be noted that the average density of the Earth is 5520 kg m^{-3}.

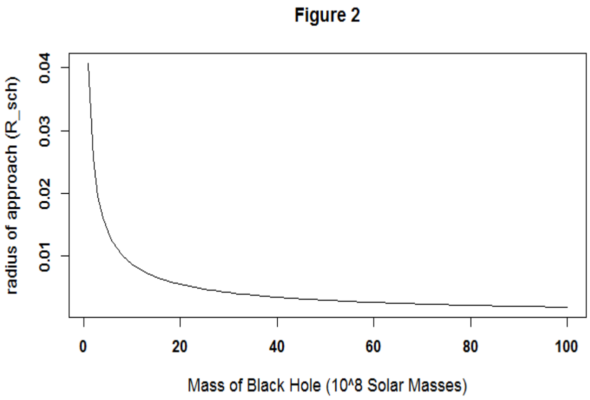

Figure 2. Plots the distance of closest approach for the Saturn V rocket in Schwarzschild radii against the mass of the SMBH in 100 million solar masses

Discussion and Conclusions

The results from Figure 1 show that, for a SMBH of around 100 million solar masses, as noted in [4], the density of the planet increases with a decreasing mass for the SMBH, such that the planet must be much denser to withstand the tidal forces of a smaller Black Hole. This is understandable, given that a smaller Black Hole would represent a much steeper gradient in potential, and so denser objects would be able to withstand the tidal forces.

The average density of the Earth is taken to be 5220 kg m^{-3}, and from Figure 1 it is noted that this density corresponds very closely to the value of 100 million solar masses for the SMBH. A planet of this density could withstand a much more massive SMBH should the mass have been underestimated. The planet, however, would not be able to survive SMBH masses much below the 100 million solar mass limit. Considering this, overestimates of the mass could put question to the existence of such planets.

Figure 2 shows that the minimum approach of the rocket, before tidal disruption causes structural damage, is well within the event horizon of the Black Hole, for all considered masses. The ship would therefore be able to traverse the event horizon, as depicted in the film, although without the use of theoretical engines, such as a “warp drive” as quoted in the film, the crew would not be able to escape the high gravitational field.

In conclusion, it is possible for planets to exist in orbit about a quiescent SMBH without being affected by tidal disruption caused by the massive gravitational fields. Additionally, the crew would be able to traverse the event horizon at the Schwarzschild radius without critical structural damage to their craft.

References

[1] http://www.insidescience.org/blog/2014/11/14/were-online-critics-too-harsh-interstellars-science, B.P. Stein, Edited 14/11/2014 Accessed 19/11/2014

[2] http://www.boeing.com/boeing/history/boeing/saturn.page, Boeing, Accessed 19/11/2014

[3] http://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html, Dr D.R Williams, Edited 1/7/2013, Accessed 19/11/2014

[4] http://www.space.com/27701-interstellar-movie-science-black-holes.html, M. Wall, Edited 10/11/2014, Accessed 19/11/2014

[5] Ming, M.N., 2011, “Mars 12: Destination Mars: When Dreams Become Reality”, Ng Zhan Ming, Malaysia, Page 135