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P1_1 "Prepare for Titanfall"

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Abstract

In the popular digital entertainment title *Titanfall*, "titans", or large metal exoskeletons are dropped onto the battlefield on Earth, from a region in low earth orbit (LEO) at approximately 160 km. From start to finish, this process takes approximately 10 seconds in-game, subsequently requiring an average velocity of around $1.6\text{x}10^4$ ms^{-1}. This paper aims to incorporate the often neglected aspect of quadratic drag into the dynamical calculations in order to show that the time required for this movement is closer to 223.2 seconds once the correct atmospheric impacts have been accounted for.

Introduction and Theory

Objects incident to the Earth from orbits significantly outside the main body of the atmosphere typically undergo two distinct phases of movement. The first is manifest as freefall without drag from their origin to an altitude of approximately 12km within which, 80 – 90% of the atmosphere is suspected to lie [1]. Upon reaching this altitude the object will lose the vast majority of its speed through atmospheric interactions, effectively resetting its speed to 0 and letting freefall with drag take the object from 12 km to the ground.

The approach to the temporal calculations will take the form of regular equations of motion, the relevant one in the first phase of movement shown below in equation (1):

$$\Delta y = v_0 t + \frac{1}{2} a t^2, \quad (1)$$

where *Δy*, v_0, *t* and *a* denote height change, initial velocity, time and acceleration.

Application

Using Δy as (160-12)km, $v_0$as 0 and *a* as 9.81 ms^{-2}, we can trivially calculate with equation (1), that, falling from stationary, it would take the "titan" 173.7 seconds to reach the outer edge of the second phase. In the second phase, Newton's equation of motion modified for quadratic drag will be used. Depending on the speed of the object, drag is either directly proportional to velocity at low speeds, or at higher speeds, proportional to the square of velocity [2]. As this paper provisionally requires the "titan" to be moving at considerable speeds, our approach will consist of the quadratic relation shown in equation (2):

$$m\frac{d\boldsymbol{v}}{dt} = m\boldsymbol{g} - C_d \boldsymbol{v}^2, \quad (2)$$

where *m, v,* $\boldsymbol{g}$ and C_d denote the mass, velocity, gravitational acceleration and drag coefficient respectively. In order to solve for *v*, and eventually raw displacement, equation (2) is rearranged and integrated into the form shown below in equation (3):

$$\int_0^y \frac{d\boldsymbol{v}}{1-\frac{\boldsymbol{v}^2}{v_t^{\,2}}} = \boldsymbol{g}\int_0^y dt, \quad (3)$$

where v_t has been substituted in place of $(m\boldsymbol{g}/C_d)^{0.5}$, and the limits of integration represent the altitude of phase two (y) and the ground (0).

Changing variable here to $v/v_t = z$, the integral on the left hand side becomes one of standard form that is known as $tanh^{-1}(z)$. Once this is re-inserted into equation (3), with the knowledge that the elements of integration have also been modified by the change in variable, the result of equation (4) follows [3]:

$$\boldsymbol{v}(t) = v_t \tanh(\frac{gt}{v_t}). \quad (4)$$

This shows, as expected, asymptotic behaviour toward the terminal velocity from zero shown in figure (1):

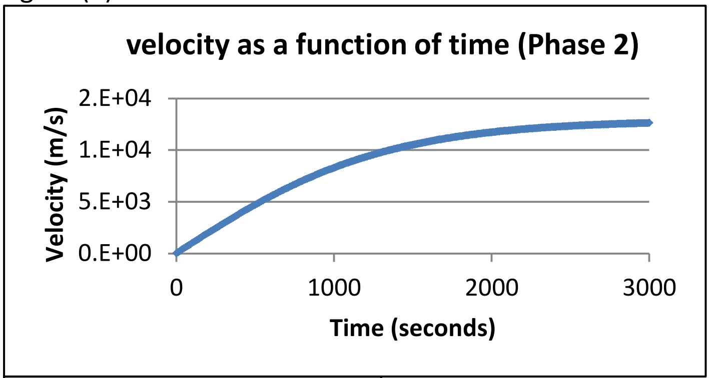

Figure 1: A representation of equation (4)

As a result of this, a further integration is necessary in order to find the time required for the “titan” to fall from is approximate phase two altitude of 12,000m to the ground. We start this process by integrating equation (4) through methods considered extraneous to the objective of this paper. Our result is an expression for the altitude as a function of time shown in equation (5) [3]:

$$\boldsymbol{y}(t) = \frac{v_t{}^2}{\boldsymbol{g}} Ln\left[Cosh\left(\frac{gt}{v_t}\right)\right]. \quad (5)$$

Returning to equation (2), intuitively, the terminal velocity $\boldsymbol{v}_t$ can be found simply by setting the left hand side to zero, and rearranging for $\boldsymbol{v}$. Once m, $\boldsymbol{g}$, and C_d have been substituted in, we find that the maximum velocity of the falling “titan” in phase two of its movement is: $\boldsymbol{v}_t = 12.87_{x10}{}^{3}$ ms^{-1}. Substituting this into equation (5), rearranging for t and using 12,000m for y we find that it is expected to take 49.5 seconds for the entirety of phase 2.

As clarification, the authors must highlight the largest of the approximations made in this paper, that which pertains to the calculation of the drag coefficient: equation (6) shows the typical bespoke method of calculating this [4]:

$$C_d = {}^{m|\boldsymbol{g}|}\Big/_{\frac{1}{2}\rho|\boldsymbol{v}^2|A}, \quad (6)$$

using ρ *and* A as atmospheric density and drag susceptible area respectively. The approximation forces the atmospheric density to remain constant at 1 kgm^{-3} throughout the second phase of movement, the drag coefficient be calculated using a mass value of 5,400 kg and an area 1.28 m^2, (Explicit calculations considered irrelevant to the paper’s purpose) and a fixed velocity figure of $1.6_{x10}{}^{4}$ ms^{-1}, which is based on a simple distance/time calculation using LEO altitude and the 10 seconds seen in-game. As a result of this, the drag coefficient is calculated to be $C_d = 3.2_{x10}{}^{-4}$.

Conclusion

Subsequently, and unsurprisingly, in contrast to the 10 seconds quoted in-game, the actual flight time of the “titan” would comprise of phase one: 173.7 seconds and phase two: 49.5 seconds, totalling 223.2 seconds. However, in the context, artistic license is expected, as conforming to the figures calculated here would severely detract from gameplay.

References

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