# Journal of Physics Special Topics

## A2_6 Look at the size of that!

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Nov 20, 2014.

**Abstract**
Further to the paper titled "Terraforming Mars: Creating a Magnetic Field" we investigate the size of the magnetopause that would be created by giving Mars a dipolar magnetic field. It is found that if the magnetic field at the equator is the same as that on Earth ($\simeq 31,000$nT) then the stand-off distance of the magnetopause would be roughly 37000km.

### Introduction

Previously we discussed how a magnetic field could be created on Mars by using a loop of copper around the equator and putting a current through it. [1] Now we move on to working out what size magnetosphere this would actually create by calculating the stand-off distance between the solar wind and the magnetopause.

### Discussion

As Mars is further away from the Sun than the Earth, the flux from the solar wind will be lower. We calculated the solar wind mass density by using the fact that,

$$r^2 \rho v = constant, \tag{1}$$

where; $r$ is the orbital radius of the planet, $\rho$ is the solar wind mass density, and $v$ is the velocity of the solar wind. Using values of $5 \times 10^6$m$^{-3}$ for the solar wind mass density at Earth, the orbital radius of the Earth being 1AU ($1.8 \times 10^8$m), the orbital radius of Mars being 1.52 AU, and $v$ being assumed to be constant at 500kms$^{-1}$, we can calculate a value for the solar wind mass density at Mars by using Eqn.(1) to derive,

$$\rho_m = \frac{r_E^2 \rho_E}{r_m^2}. \tag{2}$$

The subscripts $m$ and $E$ denote Mars and Earth respectively, and this gives a value for $\rho$ at Mars of $3.62 \times 10^{-21}$kgm$^{-3}$.

Knowing the mass density of the solar wind at Mars we can now calculate the pressure of the solar wind using,

$$P_{sw} = \rho v^2, \tag{3}$$

where; $P$ is the pressure and the subscript $sw$ denotes the solar wind.

To find the size of the magnetopause we must find the point at which the pressure of the solar wind is equal to the pressure of the magnetopause, known as the stand-off distance. The pressure of the magnetopause is given by,

$$P_{mp} = \frac{B_{mp}^2}{2\mu_0}, \tag{4}$$

where; $B$ is the magnetic field strength, the subscript $mp$ denotes the magnetopause, and $\mu_0$ is the permeability of free space.

Equating Eqn.(3) and Eqn.(4) we can determine the magnitude of the magnetic field at the magnetopause by,

$$B_{mp} = \sqrt{2\rho v^2 \mu_0}. \tag{5}$$

Using $\rho$ as determined earlier we can calculate that the magnetic field strength at the magnetopause of Mars is 47.7nT.

As $B_{mp}$ is $2B_{dipole}$ [3] it is then possible to find the magnetic field strength at the magnetopause in terms of the magnetic field strength at the equator through,

$$B_{mp} = 2\left(B_{eq}(1 + 3\cos^2(\theta))^{\frac{1}{2}} \left(\frac{R_p}{r_{mp}}\right)^3\right), \tag{6}$$

where; the subscript $eq$ denotes the equator, $\theta$ is the colatitude, $R_p$ is the radius of the planet, and $r_{mp}$ is the distance to the edge of the magnetopause (stand-off).

At the equator $\theta$ is 90 degrees so Eqn.(6) becomes,

$$B_{mp} = 2B_{eq} \left( \frac{R_p}{r_{mp}} \right)^3 . \tag{7}$$

Rearranging for $r_{mp}$ we get,

$$r_{mp}^3 = B_{eq} \frac{2R_p^3}{B_{mp}} . \tag{8}$$

Having calculated $B_{mp}$ earlier, using $R_p$ as the radius of Mars (3389.5km) [4], and assuming that Mars has the same magnetic field strength at its equator as the Earth does (31,000nT) [2] it is possible to use Eqn.(8) to calculate the size of the magnetopause. We found the value of $r_{mp}$ to be roughly 37,000km.

**Conclusion**

Following on from our previous paper we have now shown that the standoff distance of the magnetopause created on Mars with a magnetic field of roughly the strength of Earth's is 37,000km. This is about 67% of the size of Earth's.

**References**


[1] J. Hue, J. A. Farrow, C. Checklin and R. Miller *A2_4 Terraforming Mars: Creating a Magnetic Field*, PST 13, (2014).

[2] `http://en.wikipedia.org/wiki/Terraforming_of_Mars` accessed 20/11/2014

[3] `http://en.wikipedia.org/wiki/Magnetopause#Estimating_the_standoff_distance_to_the_magnetopause` accessed 20/11/2014

[4] `http://en.wikipedia.org/wiki/Mars` accessed 20/11/2014.