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# P4_4 The Lunar Recession

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**Abstract**
This article investigates the Moon's recession from the Earth. The semi-major axis at which the Moon will become tidally locked with the Earth is calculated, and is found to be 558,000 km. It was found that the period of the Earth-Moon system would be about 48 current days long at this time. Using the Moon's current rate of recession, it was estimated that this process would happen in about 4.6 Gyr.

**Introduction**
The orbit of the Moon around Earth is not fixed; it is gradually receding from us. This is because of the tidal bulges raised by the Moon on Earth. The Earth spins faster than the Moon orbits, but because the bulges oppose the Earth's rotation, there is a resulting friction between the rotating Earth and the water. This causes the bulges to be displaced with respect to the line joining the centres of the Earth and Moon, in the direction of the Earth's rotation. Since the Moon opposes this displacement, there will be a breaking torque exerted on the Earth's rotation by the Moon. Hence, the Earth's own spin is slowing, so the days on Earth are gradually increasing. Due to the conservation of angular momentum of the Earth-Moon system, the Earth must impart the loss in its own angular momentum to the Moon's orbit. Hence, the Moon is being forced into a slightly larger orbit which means it is receding from the Earth. However, eventually this process will come to an end. This is because the Earth's own rotation rate will match the Moon's orbital rate, and it will therefore no longer impart any angular momentum to it. In this case, the planet and the moon are said to be tidally locked. This is a stable situation because it minimises the energy loss due to friction of the system. Long ago, the Moon's own rotation became equal to its orbital period about the Earth and so we only see one side of the Moon. This is known as synchronous rotation and it is quite common in the solar system [1]. This article will show that you can find the distance at which the Moon will have to orbit for the Moon and the Earth to have a common angular velocity, by considering the conservation of angular momentum and the balance between the centripetal and gravitational forces.

**Theory**
The Moon revolves about the Earth in an almost circular orbit, so we shall assume that the orbit is circular. The centripetal force must balance the gravitational force between the Earth and the Moon and hence we can write

$$\frac{mv^2}{r} = \frac{GMm}{r^2}, \tag{1}$$

where $G$ is the gravitational constant, $m$ is the mass of the Moon, $v$ its tangential velocity, $r$ its semi-major axis, and $M$ is the mass of the Earth. The tangential velocity can be replaced by the angular velocity, $\Omega$ , with the relation $v = r\Omega$, so equation (1) becomes

$$r^3\Omega^2 = GM = \text{constant}. \tag{2}$$

Therefore, the product of the cube of the semi-major axis and the square of the angular velocity must remain a constant, which means that if $r$ increases then $\Omega$ must decrease correspondingly.
Now, if we consider the conservation of angular momentum, the total angular momentum, $L$, of the current Earth-Moon system must remain a constant, and so we can equate the present day angular momenta of the Earth and Moon, with the angular momenta for a common angular velocity, $\Omega$, of the Earth and Moon and the Moon's new orbital radius, $r'$. Therefore,

$$L = I_0\Omega_0 + mr^2\Omega_m = mr'^2\Omega + I_0\Omega, \tag{3}$$

where $I_0$ is the moment of inertia of the Earth, $\Omega_0$ is the angular velocity of the Earth, and $\Omega_m$ is the angular velocity of the Moon. Substituting in the known values, $m = 7.35 \times 10^{22}$ kg, $M = 5.97 \times 10^{24}$ kg, the mean Earth-Moon distance $r = 3.84 \times 10^8$ m, $G = 6.67 \times 10^{-11}$ Nm$^2$/kg$^2$ [2], and the Earth's moment of inertia $I_0 = 8.01 \times 10^{37}$ kg/m$^2$ [3], it is found that $L$ is about $3.47\times10^{34}$ kgm$^2$/s.

Rearranging for $\Omega$ in (2) and replacing $r$ by $r'$,

$$\Omega = \sqrt{\frac{GM}{r'^3}}. \tag{4}$$

In the final state, the Earth's own angular momentum will be small compared to the Moon and hence we can ignore the angular momentum of the Earth in the final state. Substituting the above expression for $\Omega$ into (3), and after squaring both sides, we obtain the following

$$GMm^2r' = L^2. \tag{5}$$

Hence, substituting for $L$ into equation (5), $r'$ is found to be approximately $5.58\times10^8$ m, or 558,000 km. This is about 174,000 km further away than the current Earth-Moon distance.

Using equation (2), we can find the period, $T$, of the Earth-Moon system at this time. Substituting $\Omega = 2\pi/T$, and rearranging for the period, we obtain

$$T = \left[\frac{4\pi^2 r^3}{GM}\right]^{1/2}. \tag{6}$$

This is Kepler's third law. Hence, substituting for the mass of the Earth, $M$, and $r = r' = 5.6 \times 10^8$ m, the period of the Earth-Moon system at this time would be about 48 current days long. So the Earth would be rotating 48 times as slowly as today. Since the Earth's rotation would be at the same rate as the Moon's orbit, the Moon would appear to be at a fixed position in the sky, and hence only one side of the Earth would be able to see the Moon.
The current rate of the Moon's recession from the Earth is about 3.8 cm/yr [4]. Therefore, for the Moon to reach $5.58\times10^8$ m (a further $1.74\times10^8$ m), assuming a constant rate of recession, it would take about 4.6 Gyr. However, the rate of lunar recession is unlikely to remain constant. Indeed, palaeontological evidence seems to show that the Moon's rate of recession in the past was slower [3]. However, what is certain is that the Moon is receding and eventually the Earth and Moon will reach an equilibrium state and become tidally locked.

**Conclusion**
In summary, it was found that when the angular velocities of the Moon and Earth become equal, the semi-major axis of the Moon would be about 558,000 km. The period would, therefore, be 48 days long. This process will happen in an estimated 4.6 Gyr. However, the rate of lunar recession is estimated to have been less in the past, and thus is subject to change, meaning this is only an estimate.

**References**

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[2] P. A. Tipler, *Physics for Scientists and Engineers.* W. H. Freeman, 6th edition, (2007).
[3] K. Lambeck, *The Earth's variable rotation: geophysical causes and consequences.* Cambridge University Press, (1980).
[4] `http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html` (20/11/2014).