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P3_5 Balloon Mayhem in Gotham

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Abstract

The article investigates how high weather balloons made of helium rise before they burst and drop down to the Earth. Considering the pressure at higher altitudes, the radius of the balloon and the mass of the package we find that the balloon rises to an altitude of 37800m.

Introduction

In the television series 'Gotham' the episode called 'The Balloonman' shows an interesting scene where the villain attaches his victims to a Helium (He) filled weather balloon and later the body of the victim falls to the ground. In this paper we calculate the maximum height that the balloon will rise to, for this we shall consider the pressure at which the radius of the balloon expands to its limit and bursts.

Theory

Weather balloons are most commonly filled with Helium or hydrogen and are sent into the upper atmosphere with their scientific payloads [1]. Usually when the balloon bursts a parachute is released and the payload safely falls back down to Earth. These balloons are made of latex with a thickness (t_1) of 0.051mm, and the radius can expand massively depending on the pressure acting on it [2]. The balloon used in the television programme would need to be able to lift a minimum mass of 90kg, the average mass of a male.

Since we assume that the balloon is made of helium and is uniform and spherical, we first calculate the volume of He needed inside the balloon that is required to lift this mass. Helium can lift approximately 1.02g per litre [3], so the initial required volume must be 9000l, or 9m^3. Rearranging equation 1 we can find the radius of the balloon required.

$$V = \frac{4}{3}\pi r^3 \quad (1)$$

V_1 is the volume of the balloon at sea level, which we already determined should be at least 9m^3 and therefore r_1, the radius of the balloon equals 1.3m.

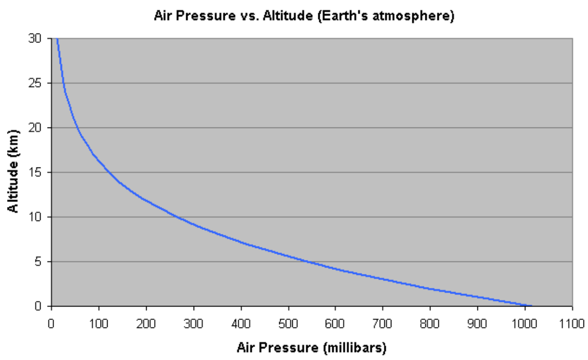


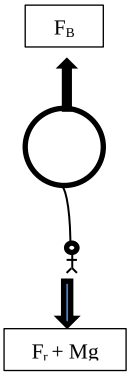
Figure 1. A graph of pressure vs Altitude for the Earth [4]

As the balloon rises and reaches higher altitudes, its radius will expand due to the decreasing atmospheric pressure, as shown in figure 1 below. The balloon will then burst when the skin thickness of the balloon reaches approximately t_2=0.0025mm [2]. In addition we assume that the temperature remains constant over this height, which is a valid assumption used by meteorologists when sending up weather balloons. To work out the size of the balloon when it pops at the high altitude, we firstly work out the pressure inside the balloon using the ideal gas equation 2. However we firstly need to work out the number of moles of helium in the balloon, we can do this using equation 3. The mass, m of the helium inside the balloon is given by the density of helium is given by ρ_{He}=0.1663 [4], multiplied by the volume of the balloon is V_1 (9m^3) and the molar mass of helium is 4.002 [5].

$$PV = nRT \quad (2)$$

$$n = \frac{m}{Molar\ mass} \quad (3)$$

Now we can work out the pressure inside the balloon P_{B1}, V_1 is the volume of a sphere, n is the number of moles of helium (n=372mols), R is the gas constant, and T is a constant temperature, 288k. Therefore the pressure inside the balloon P_{B1}=99001Pa and hence the balloon rises as it is less than atmospheric pressure. Using equation 4 we then calculate the stress of the balloon wall [3], using values of the thickness t_1 and radius r_1 of the balloon the stress equals 3.38x10^{9}Nm^{-2}.



$$Stress = \frac{Pr}{2t} \quad (4)$$

$$r = \left[\frac{3nRT}{Stress.2t_2.4\pi}\right]^{0.5} \quad (5)$$

Next to calculate the height that the balloon reaches, we need to calculate the pressure outside the balloon at this height. We start by calculating the net force acting on the balloon. As shown in figure (2) the forces on the balloon are the buoyancy force (F_B), friction (F_r), and weight (Mg); in this scenario we are neglecting friction. Initially the buoyancy force would have to be greater than the weight but as the balloon rises it reaches a terminal velocity at which $F_B=mg=\rho_{air}V_2g$. Therefore at the bursting altitude, the density of air outside this balloon is calculated using equation (6) where ρ_{air} is the density of air, V_2 is the volume of the balloon and m is the mass of the balloon, which we calculated previously.

Figure 2 shows the two forces acting on the balloon. As the balloon rises these are equal and the balloon rises at a constant speed VT, terminal velocity.

$$\rho_{air} = \frac{V_2}{m} \quad (6)$$

In equation 6 m=1.6kg, the volume equals 825m^3; therefore the density of air outside the balloon at this unknown altitude is ρ_{air}=1.95x10^{-3}kgm^{-3}.

We can finally calculate the altitude, Z, at which the balloon will pop using equation (7) below, where ρ_{air} is the density of air we just calculated, ρ_0 is the density of air at sea level, and H is the scale factor 8.5km [6]. Therefore the balloon rises to a height of 37.8km.

$$\rho_{air} = \rho_0 exp^{\frac{-Z}{H}} \quad (7)$$

Conclusion

To investigate how high a weather balloon will travel we considered its initial mass and volume before taking off and then calculated the size of the balloon that it would burst at and then what the density of the air at this height would be to calculate its height. We calculated that the balloon in this scene reaches a height of 37.8km.

References

[1] http://www.highaltitudescience.com/pages/intro-to-weather-balloons accessed on 02/11/2014
[2] http://www.srh.noaa.gov/bmx/?n=kidscorner_weatherballoons accessed on 02/11/2014
[3] http://science.howstuffworks.com/helium.htm accessed on 02/11/2014
[4] http://hypertextbook.com/facts/2002/JaneRubinshteyn.shtml accessed on 02/11/2014
[5] http://www.webqc.org/molecular-weight-of-He.html accessed on 02/11/2014
[6] http://en.wikipedia.org/wiki/Scale_height accessed on 02/11/2014