# Journal of Physics Special Topics

# P4_2 Tidal forces in the solar system

L. E. J. Evans, D. Cherrie, L. Ingram, A. Tyler

*Department of Physics and Astronomy, University of Leicester. Leicester, LE1 7RH.*

October 22, 2014.

**Abstract**
This article investigates the tidal accelerations exerted by planets on orbiting moons. The tidal acceleration was calculated, in terms of the acceleration due to gravity at the planet's surface, for several planetary systems in the solar system. A plot was created which showed that for most planetary systems, the tidal acceleration varied inversely as the cube of the semi-major axis, as theory predicted for the Earth and Moon.

**Introduction**
The tidal force arises because the gravitational force exerted on one body by another is not constant across its diameter. In the case of the Earth-Moon system the side of the Earth closest to the Moon experiences a higher gravitational attraction than the furthest side. Thus, the tidal force is a differential force. According to Newton's third law, there is an equal and opposite force exerted that results in equal tidal bulges for both the near and far sides. The displaced tidal bulges exert a net force on the Moon which accelerates it in its orbit. For this investigation, the tidal acceleration for several planetary systems in the solar system will be calculated. Let us begin by finding the equation to calculate the tidal force.

**Theory**
For simplicity let us consider only the Earth-Moon system and neglect the Sun's influence since its tidal force is less than half that of the Moon [1, p. 65]. The tidal force is defined as the difference between the gravitational force directed towards the Moon from a point on the Earth's surface closest to it along the Earth-Moon axis, and that at the centre of the Earth. The magnitude of the tidal force is approximated by assuming that the distance between the centres of the Earth and Moon is much greater than the radius of the Earth. The tidal force is thus [1, p. 64]

$$\Delta F \approx \frac{2GM_1 mR}{r^3}, \tag{1}$$

where $G$ is the gravitational constant, $r$ is the distance between the centres of the Earth and Moon, $M_1$ is the mass of the Moon, and $R$ is the radius of the Earth.
To find the tidal acceleration we can substitute for $G$ into equation (1). The gravitational force per unit mass on the surface of the Earth of mass $M_2$ is

$$g = \frac{GM_2}{R^2}. \tag{2}$$

Rearranging equation (2) for $G$ and substituting for it into equation (1), and upon dividing through by $m$ we obtain the tidal acceleration as

$$a_t = 2g\left(\frac{M_1}{M_2}\right)\left(\frac{R}{r}\right)^3. \tag{3}$$

**Results and Discussion**
We now calculate the tidal acceleration in terms of the acceleration due to gravity $g$ at the Earth's surface using equation (3). The mass of the Moon is $M_1 = 7.35\times10^{22}$ kg, the distance between the centres of the Moon and the Earth is $r = 3.84\times10^5$ km, the mass of the Earth is $M_2 = 5.97\times10^{24}$ kg, and the Earth's radius $R = 6371$ km to give $a_t$ as about $1.1\times10^{-7}g$.
For comparison, the tidal accelerations in other planetary systems in the solar system will be calculated. We make the assumption that in other planetary systems the distance between the centres of the planet and the orbiting moon is much greater than the planet's radius, as with the Earth-Moon system. This allows us to use equation (3) to calculate the tidal acceleration in terms of the acceleration due to gravity,

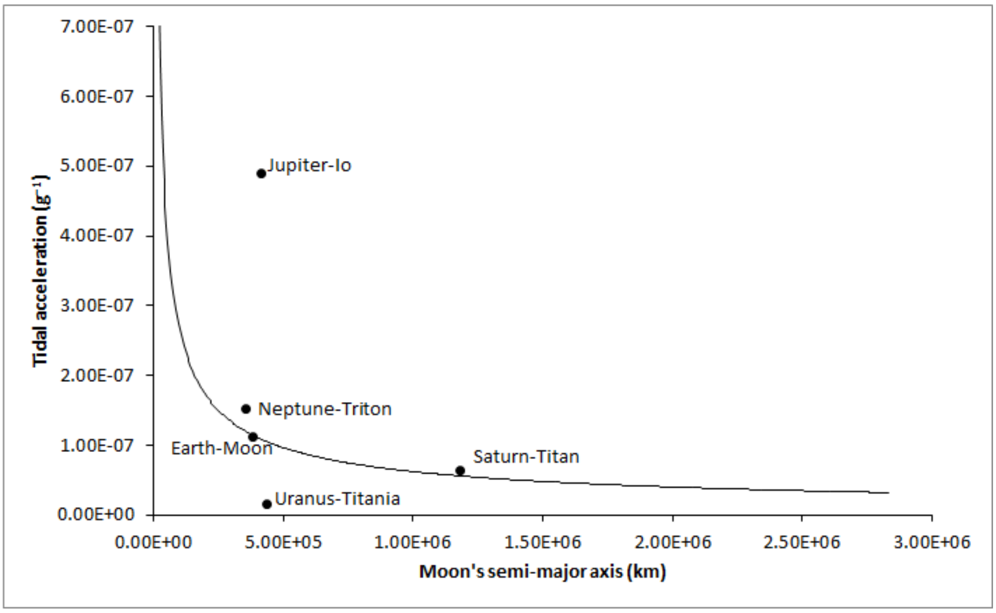


Fig. 1: A plot of the tidal accelerations in terms of the planet's gravitational acceleration for several planetary systems in the solar system. [3]

$g$, at the planet's surface. We will calculate the tidal acceleration for Jupiter and its moon Io, Saturn and its largest moon Titan, Uranus and its largest moon Titania, as well as Neptune and its largest moon Triton. Proceeding as before, the mass of the orbiting moon is $M_1$, the mass of the planet is $M_2$, the distance between the centres of the planet and the orbiting moon is $r$, and the radius of the planet is $R$. The results for the tidal accelerations are summarised by the plot in figure 1.
The tidal accelerations and hence the tidal forces are generally very small in the solar system. Despite Io being of similar size to the Moon and orbiting at a larger distance to Jupiter it does experience a larger tidal acceleration than the Moon. The deviation from the trend is most likely because the ratio of Jupiter's radius to the semi-major axis of Io is significantly larger. Figure 1 shows that most of the planet-moon systems closely follow an inverse cube relation between the semi-major axis and the tidal acceleration, as in equation (3). The tidal forces seen on Io are in fact some of the largest seen in the solar system and cause enough heating to make it very volcanically active. This is mainly due to Jupiter's enormous size and Io's close proximity to Jupiter. The forces cause Io's surface to bulge outwards by as much as 100 m which is massive in comparison to Earth, where the difference in high and low tides can be about 18 m [2]. If we bear in mind that the Earth is mostly made out of water whereas Io is solid ground then we can see why, from these calculations, Io would have a lot of heat generated inside it.

## Conclusion

By comparing a few planets together we found that the tidal accelerations are generally very small in the solar system. However, for very large planets like Jupiter with Io orbiting in close proximity, the tidal accelerations can be almost 5 times as large as the for the Earth-Moon system. Such large forces can alter the geological activity on the moon leading to major volcanic activity. The assumption made that the radius of the planet was much less than the distance between the centres of the planet and moon turned out to be a reasonable assumption, as figure 1 indicates most planets and moons follow this trend. In our calculations we neglected the gravitational effects of other orbiting moons, the frictional forces on the planet's surface, and the tidal dissipation.

## References

[1] D. T. Pugh, *Tides, Surges and Mean Sea-Level.* Wiley, (1987).
[2] `http://solarsystem.nasa.gov/planets/profile.cfm?Object=Jup_Io` accessed 22/10/2014.
[3] All planet and satellite data obtained from: `http://nssdc.gsfc.nasa.gov/planetary/planetfact.html` accessed 05/11/2014.