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A4_1 It's a-me Density!

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Abstract

This paper considers the composition of small scale planets as seen in the video game *Super Mario Galaxy*. With an approximate radius for these planets set at 50m, the required density to maintain Earth-like surface gravity is calculated and compared to known super-dense structures such as white-dwarfs. The stability of such a planet is then discussed and it is concluded that it would likely explode due to the severe imbalance of gravitational pressure to degeneracy and coulomb pressures.

Introduction

Super Mario Galaxy is a 2007 video game with an interesting take on planetary science [1]. The various planets visited in the game appear to be approximately 100m in diameter. This leads to the curvature of their surfaces being not only visible but extreme, with Mario often walking around the whole circumference of a planet in a minute or two. His movement and jumping capabilities appear the same on each planet, as well as on Earth, leading to the assumption that they all have the same surface gravity (9.81ms^{-2}). So how dense would these 'baby' planets need to be in order to generate the required gravitational force and is this theoretically possible?

Baby Planet or Dwarf Star?

To find the density of the theoretical planet, its mass first needs to be calculated. We start with the equation for the acceleration due to gravity,

$$g = \frac{GM}{r^2}. \tag{1}$$

G$= 6.67 \times 10^{-11}$ $\text{Nm}^2\text{kg}^{-2}$ is the universal gravitational constant, M is the mass of the planet, g is the acceleration due to gravity on the surface and r is the planet's radius. Assuming Earth's surface gravity g$= 9.81\text{ms}^{-2}$, and a planet radius of 50m the mass M can be calculated by rearranging eq.1,

$$M = \frac{gr^2}{G} = \frac{9.81 \times 50^2}{6.67 \times 10^{-11}} = 3.68 \times 10^{14} kg. \tag{2}$$

The density of the planet can then be found using,

$$\rho = \frac{M}{V} = \frac{3M}{4\pi r^3} = \frac{3 \times 3.68 \times 10^{14}}{4\pi \times 50^3} = 7.02 \times 10^8 kgm^{-3}, \tag{3}$$

where ρ is the density and V$=\frac{4}{3}\pi$ r^3 is the volume of the planet. Earth's density ρ_e equals $5.52 \times 10^3\text{kgm}^{-3}$ which is dramatically lower than the required density of the baby planet. Osmium is the most dense element known to man and has a density $\rho_o = 22,600\text{kgm}^{-3}$ [2]. By rearranging eq.3 for M, the mass of a solid Osmium planet of radius 50m can be obtained. The gravitational acceleration on the surface of this planet can then be found to be 0.423ms^{-2} using eq.1.

White dwarfs have a density $\rho_w \approx 10^9\text{kgm}^{-3}$ [3], which is much closer to the density of the theoretical planet and therefore a candidate for its construction. However this density is for an Earth sized white dwarf with gravity in the region 10^6ms^{-2}, found using eqs. 3 and 1. For a white dwarf to be smaller than this, its density would need to be far greater to produce the required crushing force, as discussed in the next section.

The Fate of the Planet

If a planet with the density of a white dwarf, a 50m radius, and Earth-like gravity were to be constructed what would happen?

Under these conditions, super-compression of the material needs to be considered. If confined in too small a space, elementary particles are not only affected by electric repulsion, but also by a quantum repulsion between electrons. If this quantum force is larger than electric repulsion, electrons become degenerate and exert an additional degeneracy pressure against the gravitational pressure. In white dwarfs this pressure is balanced by gravity to produce a stable body, however the baby planets would not have enough mass to have this stability as shown below.

Using the interior conditions of a known white dwarf star will give insight into the internal mechanics regarding electrons. White dwarf stars are made of carbon [3] meaning that the atomic mass of carbon can be used. The carbon atoms in Sirius B are *pressure ionised* meaning that high gravity has led to 4 outer electrons being ejected from the atoms [6].

The atomic mass of carbon is 12.0u which is equivalent to 1.99×10^{-26}kg [5], and the mass of the baby planet is 3.68×10^{14}kg, giving 1.84×10^{40} carbon atoms in the planet.

Taking 4 free electrons for each atom gives 7.38×10^{40} free electrons in the planet. Dividing this number by the volume of the planet gives a number density (ρ_N) of 2.50×10^{35} free electrons per m^3. The degeneracy pressure of free electrons is therefore given by [4];

$$P = \frac{(3\pi^2)^{2/3}\hbar^2}{5m_e}{\rho_N}^{5/3} = \frac{(3\pi^2)^{2/3} \times (1.06 \times 10^{-34})^2}{5 \times 9.11 \times 10^{-31}} \times (2.50 \times 10^{35})^{5/3} = 2.33 \times 10^{21} Nm^{-2}. \quad (4)$$

$m_e = 9.11 \times 10^{-31}$kg is the rest mass of an electron, $\hbar = 1.06 \times 10^{-34}$Js is the reduced Planck constant and ρ_N is the number density of free electrons.

The calculated degeneracy pressure is counteracted by the gravitational pressure from the mass of the planet. In the centre of the planet, the pressure due to gravity is given by [7];

$$P_g = \frac{GM^2}{8\pi r^4} = \frac{6.67 \times 10^{-11} \times (6.68 \times 10^{14})^2}{8\pi \times 50^4} = 5.74 \times 10^{10} Nm^{-2}. \quad (5)$$

This imbalance of forces is clearly untenable, and would result in the destruction of the planet.

Short-Range Gravity

Leaving aside the issue of the planet's implausibility, what affects would being on the surface of such a small planet have on Mario? Although gravity is 9.81ms^{-2} at his feet, the inverse square law shows that the gravity at his head would be noticeably different. Taking Mario's height to be 1.5m, eq.1 gives an acceleration due to gravity of 9.237ms^{-2}. The slight lack of resistance to upwards blood flow would inflate and redden the subject's face. It is possible that this is the source of Mario's baby-like complexion.

Due to the quick fall-off of gravity with height, the escape velocity on a baby planet would be low,

$$V_e = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 3.68 \times 10^{14}}{50}} = 31.3ms^{-1} = 70.1mph. \quad (6)$$

This is large enough that Mario would not simply be able to jump off the planet however it would require significantly less force to leave the surface than what is required on Earth.

Conclusion

Although a pleasant idea, none of the above could ever truly come to pass. Clearly, the degeneracy pressure far outstrips the gravitational pressure by eleven orders of magnitude. The outcome of this discrepancy is that if constructed, the planet would survive for only a very brief moment before violently destroying itself and any short plumbers who happen to be running about on its surface.

References

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