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A4_4 Deeper Than Any Elephant Has Gone Before

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Abstract
In this paper, the depth of an Olympic size swimming pool required to crush an elephant is calculated. The depth of the swimming pool required to half the volume of an elephant was calculated to be 1.06×10^6 m (1060km) equivalent to a pressure of 1.04×10^{10} Pa (approximately 100000 times that of atmospheric pressure). Although this is an overestimate, it shows the strength of bone.

Introduction
Elephants are one the largest land animals on Earth known for their sheer size and strength. This theoretical situation uses a simplified model of an elephant to determine an upper limit on how deep an Olympic swimming pool would have to be crush an elephant to half of its volume.

Crushing the Elephant
There are a couple of assumptions that were made to simplify our calculations. The elephant is made of a mixture of tissue, ivory and bone among other things, however, to calculate a maximum depth (overestimate) we assume that to the elephant is made entirely of bone. The other assumption is that water is treated as incompressible.

To find the depth needed to crush the elephant we first had to calculate the pressure required to half the elephants volume. This was found using the following equation:

$$K = -V\frac{dP}{dV}, \tag{1}$$

Where K is the bulk modulus of bone (1.5×10^{10} Pa) [1], V is the volume of the elephant and dP/dV is the differential of pressure with respect to volume [2]. This equation is rearranged to separate the pressures and volumes resulting in the following:

$$-K\int_{V_1}^{V_2}\frac{1}{V}dV = \int_{P_1}^{P_2}dP, \tag{2}$$

Where K is as before, V_1 is initial volume of the elephant, V_2 is the final volume of the elephant equal to $V_1/2$, P_1 is the initial pressure (1 atmosphere or 1.01×10^5 Pa) [3] and P_2 is the final pressure. Integrating each side gives the following result:

$$-Kln\left(\frac{V_2}{V_1}\right) = P_2 - P_1\,. \tag{3}$$

This can be rearranged to find P_2 by substituting the values in for the other terms giving the following equation:

$$P_2 = -1.5 \times 10^{10} \times \ln\left(\frac{1}{2}\right) + 1.01 \times 10^5 = 1.04 \times 10^{10} Pa\,. \tag{4}$$

To calculate how deep the pool must be to achieve this pressure Bernoulli's equation was applied to the scenario. Bernoulli's equation is as follows:

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 + P_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + P_2\,, \tag{5}$$

Where ρ is the density of water (1000kgm^{-3}), v_1 and v_2 are the velocities at the surface of the pool and at the depth required to crush the elephant respectively, h_1 is the height of surface of the pool and h_2 is the final depth of the pool and the pressures are as before [4].

It is assumed that v_1 and v_2 are 0 and $h_1 - h_2$ is renamed Δh, simplifying the equation. It is then rearranged to find Δh giving:

$$\Delta h = \frac{P_2 - P_1}{\rho g}\,, \tag{6}$$

Where all of terms are as described before [5]. Substituting in the pressures, gravitational acceleration and density of water into this equation give the following:

$$\Delta h = \frac{1.04\times10^{10} - 1.01\times10^{5}}{1000\times 9.81}\,. \tag{7}$$

This calculation gives a depth of 1.06×10^6m (1060km). This number does appear to be on the large side, the deepest point on Earth being the Marianas Trench with a maximum depth of 10994m (≈11km) where the pressure is approximately 1.1×10^8 Pa [6]. Again the value calculated is based on an elephant consisting entirely of bone, which has a very large bulk modulus.

Conclusion

The calculated depth it would take to crush an elephant to half of its volume would be 1060km. This number is overestimate based on the assumptions we have made but gives an idea on bone's ability to withstand pressures far greater than our atmosphere. A more accurate value for the depth would need contributions from the volume of organic matter and ivory within the elephant and could be calculated in a future investigation.

References

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[6] http://geology.com/records/deepest-part-of-the-ocean.shtml; last viewed on 15/10/2014