

P3_5 I Will Blow Your House In

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Abstract

The Three Little Pigs fable describes the disastrous fate of three pigs houses caused by a wolf with well-endowed lungs. This paper models the pigs houses as a sealed box over which wind blows with constant velocity. We consider the negative pressure created by this wind, and calculate the velocity required to blow off a straw, wood, and slate roof to be $4829ms^{-1}$, $6662ms^{-1}$ and $8301ms^{-1}$ respectively. We then compared these to typical tornado velocities.

Introduction

The well-known fable *The Three Little Pigs* dates back centuries. In the fable a wolf attempts to blow down the three little pigs' houses, each made out of different materials; straw, sticks and brick. There are various ways in which wind could knock down a house; in this report we shall focus on the forces required to lift the roof off of the house.

Theory

Within this report a house is considered to be a sealed cube composing of a house structure and a roof component. In order for the roof to become detached from the house structure, the force created by the wolf's blow must be greater than the force keeping the roof attached to the house. To calculate the force required to remove the roof we shall start with Bernoulli's equation. For outside the house this can be calculated using

$$\frac{P}{\rho} + gz + \frac{v^2}{2} = const. \quad (1)$$

where P is the pressure, ρ is the density of fluid, g is the acceleration due to gravity, z is the vertical displacement and v is the fluid velocity. This will be constant along any streamline outside the house. If it assumed that when $v = 0$, $P = P_0$ then where $v \neq 0$,

$$P = P_0 - \frac{1}{2}\rho v^2, \quad (2)$$

where we consider P_0 to be the atmospheric pressure and also the pressure inside the house.

If we then calculate the difference of pressure between the inside and outside of the house,

$$\Delta P = P_0 - P = \frac{1}{2}\rho v^2, \quad (3)$$

where ΔP is the difference in pressure between inside and outside the house. Equation (3) can then be used to find the Force required to remove the roof off of the house giving

$$\Delta PA = F = \frac{1}{2}\rho Av^2, \quad (4)$$

where A is the total area of the roof.

For the roof to become detached from the house structure, $F > Mg + P_R A_p$, where M is the mass of the roof, P_R is the modulus of rupture and A_p is the total contact area between the roof and house walls. The modulus of rupture of a material is the pressure needed to break a material due to stress and strain; we will assume that the house and roof are made from a single piece of the same material. Hence combining this inequality with equation (4) and rearranging, we can calculate the minimum wind velocity required to detach the roof

$$v_{min} = \sqrt{\frac{2Mg + 2P_R A_p}{\rho A}}. \quad (5)$$

Discussion

We shall consider three different materials for the pigs houses; straw based MDF, red pine, and slate. For all of these cases, the air density, ρ , is $1.29kgm^{-3}$ [?]; we will assume the roof has an area, A , of $4m^2$ and the roof to house contact area A_p is $0.76m^2$ assuming four walls of $10cm$ thickness material. The moduli of rupture are $P_{R;MDF} = 21.02MPa$ [?], $P_{R;P} = 40MPa$ [?] and $P_{R;S} = 62.1MPa$ [?]; where subscript *MDF* refers to straw based MDF, subscript *P* refers to red pine and subscript *S* refers to slate. If we assume that the roofs are flat, we can estimate the mass of the roof. Assuming the roof material thickness is $0.02m$, with house dimensions of $2m \times 2m$, the total volume of the roof material would be $0.08m^3$. Hence the mass, M , will be this volume multiplied by the material density; where the density of straw based MDF is $750kgm^{-3}$ [?], the density of red pine is $507kgm^{-3}$ [?], and the density of slate is $2650kgm^{-3}$ [?]. Inputting these values into equation (4) for each of the different materials we can calculate the required minimum wind velocities. This yields required minimum wind velocities of $4829ms^{-1}$,

$6662ms^{-1}$ and $8301ms^{-1}$ for straw based MDF, red pine and slate, respectively.

Conclusion

The values of v calculated above indicate that the wolf would have to blow at extremely high speeds to detach the roof of these houses; considerably faster than the strongest tornados. For example for the straw-based MDF, the value of v calculated is equivalent to 34 times the speed of a grade 6 tornado; which in itself is highly unlikely [?]. Hence we suspect it is not possible for a wolf to be able to produce these wind speeds. We suspect these values are overestimates as the attachment between structure and roof would not be as strong as suggested. Furthermore, we have used the stress and strain break of a material as part of our force equation, meaning the material would have to pass the allowable stresses to break. This makes the force required to do this extremely high, possibly the reason for the high results. Further study could model the house collapsing under the force made by the wolfs breath instead of the roof being ripped off.

References

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